Shannon Meets Nash on the Interference Channel

Randall A. Berry and David N. C. Tse

Abstract

The interference channel is the simplest communication scenario where multiple autonomous users compete for shared resources. We combine game theory and information theory to define a notion of a Nash equilibrium region of the interference channel. The notion is game theoretic: it captures the selfish behavior of each user as they compete. The notion is also information theoretic: it allows each user to use arbitrary communication strategies as it optimizes its own performance. We give an exact characterization of the Nash equilibrium region of the two-user linear deterministic interference channel and an approximate characterization of the Nash equilibrium region of the two-user Gaussian interference channel to within 1 bit/s/Hz..

I. Introduction

Information theory deals with the fundamental limits of communication. In network information theory, an object of central interest is the *capacity region* of the network: it is the set of all rate tuples of the users in the network simultaneously achievable by optimizing their communication strategies. Implicit in the definition is that users optimize their communication strategies *cooperatively*. This may not be a realistic assumption if users are selfish and are only interested in maximizing their own benefit. Game theory provides a notion of *Nash equilibrium* to characterize system operating points, which are stable under such selfish behavior. In this paper, we define and explore an information theoretic Nash equilibrium region as the game

R.A. Berry is with the Dept. of Electrical Engineering and Computer Science, Northwestern University, Evanston, IL 60208, USA, email: `rberry@eecs.northwestern.edu`. D.N.C. Tse is with Wireless Foundations, University of California, Berkeley, Berkeley, CA 94720, USA, e-mail: `dtse@eecs.berkeley.edu`

Preliminary versions of this work were presented in part at the 2008 International Symposium on Information Theory and the 2009 Information Theory Workshop.

theoretic counterpart of the capacity region of a network. While the Nash equilibrium region is naturally a subset of the (cooperative) capacity region, in general not all points in the capacity region are Nash equilibria. The research question is then to characterize the Nash equilibrium region given a network and a model of the channels.

The two-user interference channel (IC) is perhaps the simplest communication scenario to study this problem. Here two point-to-point communication links interfere with each other through cross-talks. Each transmitter has an independent message intended only for the corresponding receiver. The capacity region of this channel is the set of all simultaneously achievable rate pairs (R_1, R_2) in the two interfering links, and characterizes the fundamental tradeoff between the performance achievable on the two links in face of interference.

In the cooperative setting, the users jointly choose encoding and decoding schemes to achieve a rate pair (R_1, R_2). In the game theoretic setting, on the other hand, we study the case where each user individually chooses an encoding/decoding scheme in order to maximize his own transmission rate. The two users can be viewed as playing a *non-cooperative* game, where a user's strategy is its encoding/decoding scheme and its payoff is its reliable rate. A Nash equilibrium (NE) is a pair of strategies for which there is no incentive for either user to unilaterally change its strategy to improve its own rate. These are incentive-compatible operating points. The Nash equilibrium region of the IC is the set of all reliable rate pairs each of which can be achieved at some NE. Our focus is on a "one-shot" game formulation in which each player has full information, i.e. both players know the channel statistics, the actions chosen by each player, as well as their pay-off function.

A particular IC we focus on in this paper is the two-user Gaussian IC shown in Figure 1. This is a basic model in wireless and wireline channels (such as DSL). Game theoretic approaches for the Gaussian IC have been studied before, e.g. [5]–[8]. However, there are two key assumptions in these works: 1) the class of encoding strategies are constrained to use random Gaussian codebooks; 2) the decoders are restricted to treat the interference as Gaussian noise and are hence sub-optimal. Because of these restrictions, the formulation in these works are not information-theoretic in nature. For example, a Nash equilibrium found under these assumptions may no longer be an equilibrium if users can adopt a different encoding or decoding strategy.

In this paper, we make three contributions. First, we give a precise formulation of an information theoretic NE on general ICs, where the users are allowed to use *any* encoding and

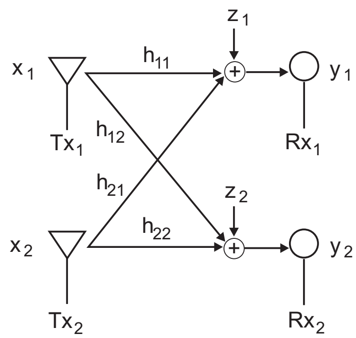

Fig. 1. Two-user Gaussian interference channel.

decoding strategies. Second, we analyze the NE region of the two-user *linear deterministic* IC [3]. This type of deterministic channel model was first proposed by [4] in the analysis of Gaussian relay networks, and the deterministic IC has been shown to be a good approximation of the Gaussian IC in [3]. For this IC, we give a simple exact characterization of the NE region. At each of the rate pairs in the NE region, we provide explicit coding schemes that achieve the rate pair and such that no user has any incentive to deviate to improve its own rate. Somewhat surprisingly, we find that in all cases, there are always Nash equilibria which are *efficient*, i.e., they lie on the maximum sum-rate boundary of the capacity region. In particular, for channels with symmetrical channel gains, the symmetric rate point on the capacity region boundary is always a Nash equilibrium. Our third contribution is to use these insights to approximate the NE region of the Gaussian IC to within 1 bit/s/Hz. This result parallels the recent characterization of the (cooperative) capacity region of the same channel to within 1 bit [1].

II. Problem Formulation

Let us now formally define the communication situation for general interference channels. In subsequent sections, we will specialize to specific classes of interference channels.

Communication starts at time 0. User i communicates by coding over blocks of length N_i symbols, $i = 1, 2$. Transmitter i sends on block k information bits $b_{i1}^{(k)}, \ldots, b_{i,L_i}^{(k)}$ by transmitting a codeword denoted by $\mathbf{x}_i^{(k)} = [\mathbf{x}_i^{(k)}(1), \ldots, \mathbf{x}_i^{(k)}(N_i)]$. All the information bits are equally probable and independent of each other. Receiver i observes on each block an output sequence through the interference channel, which specifies a stochastic mapping from the input sequences of user 1 and 2 to the output sequences of user 1 and 2. Given the observed sequence $\{\mathbf{y}_i^{(k)} =$

$[\mathbf{y}_i^{(k)}(1), \ldots, \mathbf{y}_i^{(k)}(N_i)], k = 1, 2, \ldots, \}$, receiver i generate guesses $\hat{b}_{i\ell}^{(k)}$ for each of the information bit. Without loss of generality, we will assume that each receiver i performs maximum-likelihood decoding on each bit, i.e. chooses $\hat{b}_i^{(k)}$ that maximizes the *a posterior probability* of the observed sequence $\mathbf{y}_i^{(1)}, \mathbf{y}_i^{(2)}, \ldots$ given the transmitted bit $b_{i\ell}^{(k)}$.

Note that the communication scenario we defined here is more general than the one usually used in multiuser information theory, as we allow the two users to code over different block lengths. However, such generality is necessary here, since even though the two users may agree *a priori* on a common block length, a selfish user may unilaterally decide to choose a different block length during the actual communication process.

A *strategy* s_i of user i is defined by its message encoding, which we assume to be the same on every block and involves:

- the number of information bits B_i and the block length N_i of the codewords,
- the codebook $\mathcal{C}_i$, the set of codewords employed by transmitter i,
- the encoder $f_i : \{1, \ldots, 2^{B_i}\} \times \Omega_i \to \mathcal{C}_i$, that maps on each block k the message $m_i^{(k)} := (b_{i1}^{(k)}, \ldots b_{i,B_i}^{(k)})$ to a transmitted codeword $\mathbf{x}_i^{(k)} = f_i(m_i^{(k)}, \omega_i^{(k)}) \in \mathcal{C}_i$,
- the rate of the code, $R_i(s_i) = B_i/N_i$.

A strategy s_1 of user 1 and s_2 of user 2 jointly determines the average bit error probabilities $p_i^{(k)} := \frac{1}{B_i}\sum_{\ell=1}^{B_i} \mathcal{P}(\hat{b}_{i\ell}^{(k)} \neq b_{i\ell}^{(k)})$, $i = 1, 2$.[1] Note that if the two users use different block lengths, the error probability could vary from block to block even though each user uses the same encoding for all the blocks. However, if they use the same block length, then the error probability is the same across the blocks for a user, which we will denote by p_i for user i.

The encoder of each transmitter i may employ a stochastic mapping from the message to the transmitted codeword; $\omega_i^{(k)} \in \Omega_i$ represents the randomness in that mapping. We assume that this randomness is independent between the two transmitters and across different blocks. Furthermore, we assume that each transmitter and its corresponding receiver have access to a source of *common randomness*, so that the realization $\omega_i^{(k)}$ is known at both transmitter i and receiver i, but not at the other receiver or transmitter.[2]

[1] Average bit error probabilities are more meaningful than codeword error probabilities in a setting, such as ours, where users can vary the blocklength they are using.

[2] Such common randomness is not needed for many of the results in the paper, but allowing for it simplifies our presentation.

For a given error probability threshold $\epsilon > 0$, we define an ϵ-interference channel game as follows. Each user i chooses a strategy s_i, $i = 1, 2$, and receives a pay-off of

$$\pi_i(s_1, s_2) = \begin{cases} R(s_i), & \text{if } p_i^{(k)}(s_1, s_2) \leq \epsilon,\ \forall k, \\ 0, & \text{otherwise.} \end{cases}$$

In other words, a user's pay-off is equal to the rate of the code provided that the probability of error is no greater than ϵ. A strategy pair (s_1, s_2) is defined to be $(1-\epsilon)$-*reliable* provided that they result in an error probability $p_i(s_1, s_2)$ of less than ϵ for $i = 1, 2$. An $(1-\epsilon)$-reliable pair of strategies is said to achieve the rate-pair $(R(s_1), R(s_2))$.

For an ϵ-game, a strategy pair (s_1^*, s_2^*) is a Nash equilibrium (NE) if neither user can unilaterally deviate and improve their pay-off, i.e. if for each user $i = 1, 2$, there is no other strategy s_i such that[3] $\pi_i(s_i, s_j^*) > \pi_i(s_i^*, s_j^*)$. If user i attempts to transmit at a higher rate than what he is receiving in a Nash equilibrium and user j does not change her strategy, then user i's error probability must be greater than ϵ.

Similarly, a strategy pair (s_1^*, s_2^*) is an η-Nash equilibrium[4] (η-NE) of an ϵ-game if neither user can unilaterally deviate and improve their pay-off by more than η, i.e. if for each user i, there is no other strategy s_i such that $\pi_i(s_i, s_j^*) > \pi_i(s_i^*, s_j^*) + \eta$. Note that when a user deviates, it does not care about the reliability of the other user but only its own reliability. So in the above definitions (s_i, s_j^*) is not necessarily $(1-\epsilon)$-reliable.

Given any $\bar{\epsilon} > 0$, the capacity region $\mathcal{C}$ of the interference channel is the closure of the set of all rate pairs (R_1, R_2) such that for every $\epsilon \in (0, \bar{\epsilon})$, there exists a $(1-\epsilon)$-reliable strategy pair (s_1, s_2) which achieves the rate pair (R_1, R_2). The *Nash equilibrium region* $\mathcal{C}_{\text{NE}}$ of the interference channel is the closure of the set of rate pairs (R_1, R_2) such that for every $\eta > 0$, there exists a $\bar{\epsilon} > 0$ (dependent on η) so that if $\epsilon \in (0, \bar{\epsilon})$, there exists a $(1-\epsilon)$-reliable strategy pair (s_1, s_2) that achieves the rate-pair (R_1, R_2) and is a η-NE. Clearly, $\mathcal{C}_{\text{NE}} \subset \mathcal{C}$.

First, we make a few comments about the definition of $\mathcal{C}_{\text{NE}}$. In this definition, the parameter $\bar{\epsilon}$ is introduced so that $(1-\epsilon)$-reliable strategy pairs need only exist for "small enough" values of ϵ. In the definition of the capacity region for the interference channel this constraint is not

[3]In this paper, we use the convention that j always denotes the other user from i.

[4]In the game theoretic literature, this is often referred to as an ϵ-Nash equilibrium or simply an ϵ-equilibrium for a game [9, page 143].

needed, i.e. the region is equally well defined by requiring the given conditions to hold for any $\epsilon > 0$ (since, clearly if a pair of strategies are $(1-\epsilon)$-reliable, they are also $(1-\tilde{\epsilon})$-reliable for all $\tilde{\epsilon} > \epsilon$). However, when defining $\mathcal{C}_{\text{NE}}$, this condition is important. In particular a pair of strategies can be an η-NE for an ϵ-game, but not an η-NE for an $\tilde{\epsilon}$-game for $\tilde{\epsilon} > \epsilon$, since increasing the error probability threshold enlarges the set of possible deviations an agent may make. As an extreme example, consider the case where $\epsilon = 1$, in which case each agent can achieve an arbitrarily high pay-off regardless of the action of the other user and so no η-NE exists. Thus, if we required our definition to hold for any $\epsilon > 0$, $\mathcal{C}_{\text{NE}}$ would be empty.

Next, we turn to the use of η-NE in the definition. A more natural approach would be to instead simply use NE. In other words, define $\mathcal{C}_{\text{NE}}$ to be the closure of the rate pairs (R_1, R_2) such that for any ϵ small enough, that there exists a $(1-\epsilon)$-reliable strategy pair (s_1, s_2) which achieves the rate-pair (R_1, R_2) and is a NE of a ϵ-game. The difficulty with this is that to determine such a NE essentially requires one to find a particular scheme that achieves the optimal rate for a given non-zero error probability. Finding such a scheme that is extremely difficult and in general an open problem.[5] By introducing the slack η, these difficulties are removed. Moreover, since we require that this definition hold for all $\eta > 0$, this slack can be made arbitrarily small.

Finally, we would like to comment on the use of different block lengths in our definitions. First, it can argued that if there is a $(1-\epsilon)$-reliable strategy pair (s_1, s_2) that achieves a rate pair (R_1, R_2) using codes of block lengths N_1, N_2, then there exists a $(1-\epsilon)$ strategy pair that achieves the same rate pair but with each user using the same block length. This follows by considering using "super-blocks" of length N, where N is the least common multiple of N_1 and N_2. Over these super-blocks the users can be viewed as using two equal-length codes. The error probabilities, being the average bit error probabilities now across longer blocks, remain less than ϵ. This means that in computing the capacity region $\mathcal{C}$, we can without loss of generality consider only strategies in which both users use the same block lengths. Also, in the Nash equilibrium definitions, we can without loss of generality assume that in the nominal strategy, the two users use the same block length (although each user is allowed to deviate using another strategy of a different block length.).

[5]Moreover, it is not even clear if there exists such a scheme, i.e. a scheme that achieves the supremum of the rates over all $1-\epsilon$ reliable schemes.

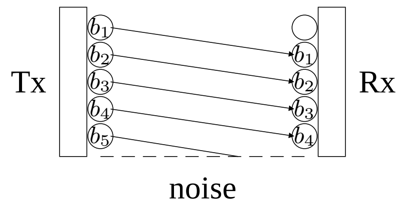

Fig. 2. The deterministic model for the point-to-point Gaussian channel. Each bit of the input occupies a signal level. Bits of lower significance are lost due to noise.

III. The Linear-deterministic IC

A. Deterministic Channel Model

Let us now focus on a specific interference channel model: a linear deterministic channel model analogous to the Gaussian channel. This channel was first introduced in [4]. We begin by describing the deterministic channel model for the point-to-point AWGN channel, and then the two-user multiple-access channel. After understanding these examples, we present the deterministic interference channel.

Consider first the model for the point-to-point channel (see Figure 2). The real-valued channel input is written in base 2; the signal—a vector of bits—is interpreted as occupying a succession of levels:

$$x = 0.b_1 b_2 b_3 b_4 b_5 \dots .$$

The most significant bit coincides with the highest level, the least significant bit with the lowest level. The levels attempt to capture the notion of *signal scale*; a level corresponds to a unit of power in the Gaussian channel, measured on the dB scale. Noise is modeled in the deterministic channel by truncation. Bits of smaller order than the noise are lost. Note that the number of bits above the noise floor correspond to $\log_2 \mathsf{SNR}$, where SNR is the signal-to-noise ratio of the corresponding Gaussian channel.

The deterministic multiple-access channel is constructed similarly to the point-to-point channel (Figure 3). To model the super-position of signals at the receiver, the bits received on each level are added *modulo two*. Addition modulo two, rather than normal integer addition, is chosen to make the model more tractable. As a result, the levels do not interact with one another.

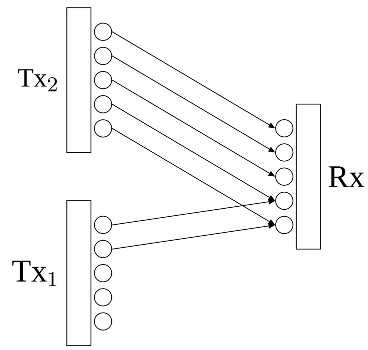

Fig. 3. The deterministic model for the Gaussian multiple-access channel. Incoming bits on the same level are added modulo two at the receiver.

We proceed with the deterministic interference channel model (Fig. 4). There are two transmitter-receiver pairs (links), and as in the Gaussian case, each transmitter wants to communicate only with its corresponding receiver. The signal from transmitter i, as observed at receiver j, is scaled by a nonnegative integer gain $a_{ji} = 2^{n_{ji}}$ (equivalently, the input column vector is shifted up by n_{ji}). At each time t, the input and output, respectively, at link i are $\mathbf{x}_i(t), \mathbf{y}_i(t) \in \{0,1\}^q$, where $q = \max_{ij} n_{ij}$. Note that n_{ii} corresponds to $\log_2 \mathsf{SNR}_i$ and n_{ji} corresponds to $\log_2 \mathsf{INR}_{ji}$, where SNR_i is the signal-to-noise ratio of link i and INR_{ji} is the interference-to-noise ratio at receiver j from transmitter i in the corresponding Gaussian interference channel.

The channel output at receiver i is given by

$$\mathbf{y}_i(t) = \mathbf{S}^{q-n_{i1}}\mathbf{x}_1(t) + \mathbf{S}^{q-n_{i2}}\mathbf{x}_2(t), \tag{1}$$

where summation and multiplication are in the binary field and $\mathbf{S}$ is a $q \times q$ shift matrix,

$$\mathbf{S} = \begin{pmatrix} 0 & 0 & 0 & \cdots & 0 \\ 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ \vdots & & & \ddots & \vdots \\ 0 & \cdots & 0 & 1 & 0 \end{pmatrix}. \tag{2}$$

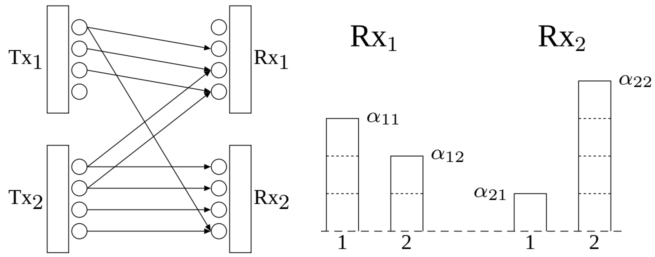

Fig. 4. At left is a deterministic interference channel. The more compact figure at right shows only the signals as observed at the receivers.

If the inputs $\mathbf{x}_i(t)$ are written as a binary number x_i, the channel can equivalently be written as

$$y_1 = \lfloor a_{11}x_1 + a_{12}x_2 \rfloor$$

$$y_2 = \lfloor a_{21}x_1 + a_{22}x_2 \rfloor ,$$

where addition is performed on each bit (modulo two) and $\lfloor \cdot \rfloor$ is the integer-part function.

In our analysis, it will be helpful to consult a different style of figure, as shown on the right-hand side of Fig. 4. This shows only the perspective of each receiver. Each incoming signal is shown as a column vector, with the highest element corresponding to the most significant bit and the portion below the noise floor truncated. The observed signal at each receiver is the modulo 2 sum of the elements on each level.

The deterministic interference channel is relatively simple, yet retains two essential features of the Gaussian interference channel: the loss of information due to noise, and the superposition of transmitted signals at each receiver. The modeling of noise can be understood through the point-to-point channel above. The superposition of transmitted signals at each receiver is captured by taking the modulo 2 sum of the incoming signals at each level, as in the model for the multiple-access channel.

B. Main Results

To begin, we give the capacity region, $\mathcal{C}$, of our two-user deterministic interference channel. This region is given by Theorem 1 in [10], which applies to a larger class of deterministic interference channels. For our model, the resulting region becomes the set of non-negative rates

satisfying:[6]

$$R_1 \leq n_{11} \tag{3}$$

$$R_2 \leq n_{22} \tag{4}$$

$$R_1 + R_2 \leq (n_{11} - n_{12})^+ + \max(n_{22}, n_{12}) \tag{5}$$

$$R_1 + R_2 \leq (n_{22} - n_{21})^+ + \max(n_{11}, n_{21}) \tag{6}$$

$$\begin{aligned} R_1 + R_2 \leq{} & \max(n_{21}, (n_{11} - n_{12})^+) \\ & + \max(n_{12}, (n_{22} - n_{21})^+) \end{aligned} \tag{7}$$

$$\begin{aligned} 2R_1 + R_2 \leq{} & \max(n_{11}, n_{21}) + (n_{11} - n_{12})^+ \\ & + \max(n_{12}, (n_{22} - n_{21})^+) \end{aligned} \tag{8}$$

$$\begin{aligned} R_1 + 2R_2 \leq{} & \max(n_{22}, n_{12}) + (n_{22} - n_{21})^+ \\ & + \max(n_{21}, (n_{11} - n_{12})^+). \end{aligned} \tag{9}$$

Our main result, stated in Theorem 1 below is to completely characterize $\mathcal{C}_{\mathrm{NE}}$ for the two-user deterministic interference channel model. This characterization is in terms of $\mathcal{C}$ and a "box" $\mathcal{B}$ in $\mathbb{R}^2_+$ given by (see Fig. 5)

$$\mathcal{B} = \{(R_1, R_2) : L_i \leq R_i \leq U_i, \forall i = 1, 2\},$$

where for each user $i = 1, 2$,

$$L_i = (n_{ii} - n_{ij})^+, \tag{10}$$

and

$$U_i = \begin{cases} n_{ii} - \min(L_j, n_{ij}), & \text{if } n_{ij} \leq n_{ii}, \\ \min((n_{ij} - L_j)^+, n_{ii}), & \text{if } n_{ij} > n_{ii}. \end{cases} \tag{11}$$

We now state our main result.

Theorem 1: $\mathcal{C}_{NE} = \mathcal{C} \cap \mathcal{B}$.

[6]The boundaries of the region in [10] is given in terms of conditional entropies that must be maximized over any product distribution on the channel inputs. For our model the optimizing input distribution for each bound is always uniform over the input alphabet. The given bounds follow.

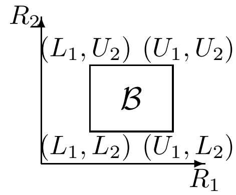

Fig. 5. An example of the box $\mathcal{B}$. The values of the four corner points are indicated in the figure.

First let us interpret the bounds L_1, L_2, U_1, U_2. The number L_i is the number of levels that user i can transmit above the interference floor created by user j, i.e. the number of levels at receiver i that cannot see any interference from user j. These are always the most significant bits of user i's transmitted signal. In the example channel in Figure 4, these correspond to the top level for transmitter 1 ($L_1 = 1$) and the top 3 levels for transmitter 2 ($L_2 = 3$). The number U_i is the number of levels at receiver i that receive signals from transmitter i but are free of interference from the top L_j levels from transmitter j. In the example channel in Fig. 4, these correspond to the top level at receiver 1 ($U_1 = 1$) and the top three levels at receiver 2 ($U_2 = 3$).

Intuitively, it is clear that at any η-NE, user i should have rate at least L_i: these levels are interference-free and user i can always send information at the maximum rate on these levels. This will create interference of maximum entropy at a certain subset of levels at receiver j and render them un-usable for user j. The rate for user j is bounded by the number of remaining levels that it can use. This is precisely the upper bound U_j. What Theorem 1 says is that any rate pair in the capacity region $\mathcal{C}$ subject to these natural constraints is in $\mathcal{C}_{\mathrm{NE}}$.

To illustrate this result, consider a symmetric interference channel in which $n_{11} = n_{22}$ and $n_{12} = n_{21}$. Let $\alpha = n_{ji}/n_{ii}$ be the normalized cross gain. Four examples of $\mathcal{C}$ and $\mathcal{B}$ corresponding to different ranges of α are shown in Fig. 6. For $0 < \alpha < \frac{1}{2}$, $\mathcal{C}_{\mathrm{NE}} = \mathcal{B}$ is a single point, which lies at the symmetric sum-rate point of $\mathcal{C}$. For $\frac{1}{2} < \alpha < \frac{2}{3}$, again $\mathcal{C}_{\mathrm{NE}} = \mathcal{B}$. $\mathcal{C}_{\mathrm{NE}}$ contains a single efficient point (the symmetric sum-rate point in $\mathcal{C}$), but now there are additional interior points of $\mathcal{C}$ which may be achieved as a Nash equilibrium.[7] For $\frac{2}{3} < \alpha < 1$, $\mathcal{C}_{\mathrm{NE}}$ is the intersection of the simplex formed by the sum-rate constraint of $\mathcal{C}$ and $\mathcal{B}$. In this case, there are multiple

[7]In a slight abuse of terminology, we say that points in $\mathcal{C}_{\mathrm{NE}}$ can be "achieved as a NE."

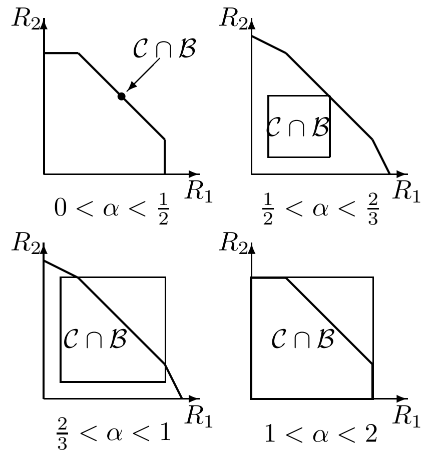

Fig. 6. Examples of $\mathcal{C}_{\mathrm{NE}} = \mathcal{C}\cap\mathcal{B}$ for a symmetric interference channel with normalized cross gain α.

efficient points; in fact, the entire sum-rate face of $\mathcal{C}$ is included in $\mathcal{C}_{\mathrm{NE}}$. For $1<\alpha<2$, $\mathcal{C}\subset\mathcal{B}$ and so $\mathcal{C}_{\mathrm{NE}}=\mathcal{C}$. For $2\leq\alpha$ (not shown) $\mathcal{C}=\mathcal{B}$ and so again $\mathcal{C}_{\mathrm{NE}}=\mathcal{C}$. Note that in all cases, the symmetric rate point is in $\mathcal{C}_{\mathrm{NE}}$.

C. Proofs

To prove Theorem 1, we first show that points outside of $\mathcal{B}$ cannot be achievable as a Nash equilibrium, formalizing the intuition discussed earlier. We will then show that all points inside $\mathcal{C}\cap\mathcal{B}$ can in fact be achieved.

1) Non-equilibrium points:

Lemma 1: If $(R_1,R_2)\in\mathcal{C}_{\mathrm{NE}}$, then $R_i\geq L_i=(n_{ii}-n_{ij})^+$ for $i=1,2$.

Proof: User i's L_i highest transmitted levels see no interference from user j's signal at receiver i. Hence, in any ϵ-game, user i can always achieve a pay-off of $R_i=L_i$ (with zero probability of error) by sending L_i uncoded bits, one on each of these levels, independent of user j's strategy. Thus for (R_1,R_2) to be obtained as an η-NE of an ϵ-game for any $\epsilon>0$, it must be that $R_i>L_i-\eta$ for all i; otherwise, user i could deviate using the above strategy and improve his pay-off by η. If $(R_1,R_2)\in\mathcal{C}_{\mathrm{NE}}$, then this inequality should hold for all $\eta>0$. Taking the limit as $\eta\to 0$, the result follows. ■

Lemma 2: If $(R_1,R_2)\in\mathcal{C}_{\mathrm{NE}}$, then for all $i=1,2$, $R_i\leq U_i$, where U_i is given in (11).

Proof: Suppose $(R_1, R_2) \in \mathcal{C}_{\text{NE}}$. Without loss of generality, let us focus on user 1 to show that $R_1 \leq U_1$.

If $n_{12} - L_2 \geq n_{11}$, then $U_1 = n_{11}$ and clearly $R_1 \leq n_{11}$, so there is nothing to prove. So in the following we can assume that

$$n_{12} - L_2 < n_{11}. \tag{12}$$

Fix an arbitrary $\eta > 0$. Given a sufficiently small $\epsilon > 0$, there exist a $(1-\epsilon)$-reliable strategy pair (s_1^*, s_2^*) achieving the rate pair (R_1, R_2) that is also a η-NE. As remarked in Section II, we can assume that in this nominal strategy pair, both users use a common block length N. Applying Fano's inequality to user 1 for the average bit error probability (see for example Theorem 4.3.2 in [11]), we get the bound, for any block k:

$$R_1 \leq \frac{I(m_1; \mathbf{y}_1 | \omega_1)}{N} + \delta \tag{13}$$

where δ depends on ϵ and goes to zero as ϵ goes to zero. Here, ω_1 denotes any randomness in $\mathbf{x}_1$, which recall is known at receiver 1. Note that we drop the block indices of the message and the signals to simplify notation. Now,

$$\begin{aligned}
& \frac{1}{N} I(m_1; \mathbf{y}_1 | \omega_1) \\
\leq \;& \frac{1}{N} I(\mathbf{x}_1; \mathbf{y}_1 | \omega_1) \\
\leq \;& \frac{1}{N} I(\mathbf{x}_1; \mathbf{y}_1) \\
= \;& \frac{1}{N} \left[H(\mathbf{y}_1) - H(\mathbf{s}_2) \right] \\
\leq \;& \max(n_{11}, n_{12}) - \frac{H(\mathbf{s}_2)}{N}
\end{aligned}$$

where $\mathbf{s}_2$ is the signal from user 2 that is visible at receiver 1. Here, the second inequality follows since $\omega_1 - \mathbf{x}_1 - \mathbf{y}_1$ forms a Markov chain. Combining this with the above inequality, we get:

$$R_1 \leq \max(n_{11}, n_{12}) - \frac{1}{N} H(\mathbf{s}_2) + \delta. \tag{14}$$

We now seek a bound on $H(\mathbf{s}_2)$. Applying Fano's inequality to user 2, we get

$$\begin{aligned}
R_2 \;&\leq\; \frac{1}{N} I(m_2; \mathbf{y}_2 | \omega_2) + \delta \\
&\leq\; \frac{1}{N} I(\mathbf{x}_2; \mathbf{y}_2) + \delta \\
&=\; \frac{1}{N} I(\mathbf{u}_2, \mathbf{v}_2; \mathbf{y}_2) + \delta
\end{aligned}$$

where $\mathbf{u}_2$ is the part of user 2's transmitted signal $\mathbf{x}_2$ which is on the top $\min(L_2, n_{12})$ levels and $\mathbf{v}_2$ is the rest. The significance of $\mathbf{u}_2$ is that it is received without interference at receiver 2 *and* is visible at receiver 1 (i.e., part of $\mathbf{s}_2$). Correspondingly, split user 2's received signal $\mathbf{y}_2 = (\mathbf{u}_2, \tilde{\mathbf{y}}_2)$. Now

$$\begin{aligned}
& I(\mathbf{u}_2, \mathbf{v}_2; \mathbf{y}_2) \\
= \; & I(\mathbf{u}_2; \mathbf{y}_2) + I(\mathbf{v}_2; \mathbf{y}_2 | \mathbf{u}_2) \\
= \; & H(\mathbf{u}_2) + I(\mathbf{v}_2; \tilde{\mathbf{y}}_2 | \mathbf{u}_2) \\
\leq \; & H(\mathbf{u}_2) + I(\mathbf{v}_2; \tilde{\mathbf{y}}_2)
\end{aligned}$$

since $\mathbf{u}_2 - \mathbf{v}_2 - \tilde{\mathbf{y}}_2$ forms a Markov chain.

Combining this with the previous equation, we get:

$$R_2 \leq \frac{1}{N} H(\mathbf{u}_2) + \frac{1}{N} I(\mathbf{v}_2; \tilde{\mathbf{y}}_2) + \delta. \tag{15}$$

Let us now consider an alternative strategy s_2' for user 2. This encoding strategy has two independent sub-codes. The first sub-code transmits uncoded bits on the top $\min(L_2, n_{12})$ levels, achieving a rate of $\min(L_2, n_{12})$ bits per symbol time with zero error. The second sub-code transmits on the remaining $n_{22} - \min(L_2, n_{12})$ levels. It codes over K blocks of length N each. Each codeword in this code has K components, each spanning N symbol times, for a total length of KN symbol times. Each codeword is chosen randomly, with i.i.d. N-length components and each component chosen from the distribution of user 2's transmit signal $\mathbf{v}_2$ under the original encoding strategy s_2^*. Note that since user 1's strategy s_1^* codes only within blocks of length N and sends independent message across different blocks, the interference from user 1 is i.i.d. across such blocks. User 2 thus faces a memoryless channel from block to block. Standard random coding arguments apply and one can show that for any $\delta_1 > 0$, there exists a large enough K such that strategy s_2' can achieve a rate of $I(\mathbf{v}_2; \tilde{\mathbf{y}}_2) - \delta_1$ bits per block and with a probability of error of less than ϵ. Thus, strategy s_2' achieves a total rate R_2' bits per symbol time reliably, where

$$R_2' = \min(L_2, n_{12}) + \frac{1}{N} [I(\mathbf{v}_2; \tilde{\mathbf{y}}_2) - \delta_1]. \tag{16}$$

By definition of η-NE, strategy s_2' cannot perform much better than s_2^*, i.e., $R_2 + \eta \geq R_2'$. Combining (16) and (15), we now have:

$$\frac{1}{N} H(\mathbf{u}_2) \geq \min(L_2, n_{12}) - \delta_1/N - \delta - \eta. \tag{17}$$

Essentially, we have shown that user 2 under strategy s_2^* must be transmitting information at maximum entropy on these $\min\{L_2, n_{12})$ levels by virtue of the fact that it forms a η-NE. Substituting this into (14) and observing that $H(\mathbf{s}_2) \geq H(\mathbf{u}_2)$, we get:

$$R_1 \leq \max(n_{11}, n_{12}) - \min(L_2, n_{12}) + \delta_1/N + 2\delta + \eta. \tag{18}$$

Under the condition (12), one can readily verify that $U_1 = \max(n_{11}, n_{12}) - \min(L_2, n_{12})$. Since η, δ and δ_1 can be chosen arbitrarily small, we have shown that $R_1 \leq U_1$. The proof is complete.

∎

2) Achievable Nash Equilibria: From Lemmas 1 and 2, it follows that $\mathcal{C}_{NE} \subseteq \mathcal{C} \cap \mathcal{B}$. In this section, we show that these two sets are in fact equal, proving Theorem 1. To do this we consider a modification of the class of Han-Kobayashi strategies presented in [2]. In these strategies, each user splits the transmitted information into two parts: private information to be decoded at only their own receiver and common information that can be decoded at both receivers. In [3] it is shown that a particular class of these strategies can achieve any point in the capacity region of the deterministic channel. The modification we make to these schemes is to allow each transmitter to include extra random bits in their common message. Next, we give some preliminary definitions related to Han-Kobayashi schemes and then formally define this class of strategies.

For a given deterministic interference channel, let $\mathcal{X}_i$ denote the input alphabet of user i, i.e. this is the set of $\max(n_{ii}, n_{ji})$. We decompose this set as the direct product $\mathcal{X}_{ic} \times \mathcal{X}_{ip}$, so that for any $\mathbf{x}_i \in \mathcal{X}_i$ can be written as $\mathbf{x}_i = (\mathbf{x}_{ip}, \mathbf{x}_{ic})$, where $\mathbf{x}_{ip}$ denotes the $(n_{ii} - n_{ji})^+$ least significant levels of $\mathbf{x}_i$, and $\mathbf{x}_{ic}$ consists of the n_{ji} most significant levels. The significance of this decomposition is that the $\mathbf{x}_{ip}$ consists of *private levels* for user i which are visible only at his receiver, while $\mathbf{x}_{ic}$ consists of *common levels* that are visible at receiver j.

We define a *randomized Han-Kobayashi* scheme for a given block-length N to be a scheme in which each each user i separates the message set $\{1, \ldots, 2^{B_i}\}$ into the direct product of a private message $\mathcal{M}_{ip}$ set containing $2^{NR_{ip}}$ messages and a common message set $\mathcal{M}_{ic}$ containing

$2^{NR_{ic}}$ messages, where $NR_{ip} + NR_{ic} = B_i$. Additionally, each user i is allowed to have a *random common message* set Ω_i consisting of $2^{NR_{ir}}$ equally likely codewords; these can be thought of as NR_{ir} random bits that the transmitter generates using the common randomness, which is shared with the corresponding receiver. The message sets are then encoded using a superposition code as follows. First the transmitter encodes the common and common random message via a map $f_{ic} : \mathcal{M}_{ic} \times \Omega_i \mapsto \mathcal{X}_{ic}^N$, where the codebook is generated using an i.i.d. uniform distribution over the common levels. Next, the transmitter encodes the private message via a map $f_{ip} : \mathcal{M}_{ip} \times \mathcal{M}_{ic} \times \Omega_i \mapsto \mathcal{X}_{ip}^N$, where for each common codeword $\mathbf{x}_{ic}$, a different private codebook is generated using an i.i.d. uniform distribution over the private levels. Here, the common codeword $\mathbf{x}_{ic}$ can be viewed as defining the cloud center and the the private codeword can be viewed as defining the cloud points. Transmitter i then sends the superposition of these two codewords. In the special case where $R_{1r} = R_{2r} = 0$, we refer to the resulting scheme as a *non-randomized Han-Kobayashi* scheme.

We call a randomized Han-Kobayashi scheme $(1-\epsilon)$-*reliable* if each user i can decode their own private and common messages with an average probability of bit error no greater than ϵ. [8]

Next we specify an achievable rate region for this class of schemes. This characterization is in terms of *modified MAC regions* for each of the two receivers. Specifically, the modified MAC region at receiver i, $\mathcal{R}_i^m$, is the set of rates $(R_{ic}, R_{ir}, R_{ip}, R_{jc}, R_{jr}, R_{jc})$ that satisfy:

$$\begin{aligned} R_{ic} + R_{ip} + R_{jc} + R_{jr} &\leq \max(n_{ii}, n_{ij}) \\ R_{ip} + R_{jc} + R_{jr} &\leq \max(n_{ij}, (n_{ii} - n_{ji})^+) \\ R_{ip} &\leq (n_{ii} - n_{ji})^+ \\ R_{ic} + R_{ip} &\leq n_{ii} \end{aligned} \tag{19}$$

The modified MAC region is derived by considering the MAC channel at receiver i consisting of three transmitters one corresponding to user i's own common message, one corresponding to user i's own private messages, and one corresponding to the combination user j's common and common random messages. Here, user i's own common random signal can be ignored since it is known at receiver i and so can be removed. Likewise, user j's private message can be

[8]This is slight strengthening of the previous definition of a reliable strategy, which only required the overall average bit error probability to be no greater than ϵ. Hence, an agent's pay-off in a ϵ-game under a $(1-\epsilon)$-reliable Han-Kobayashi scheme is again their rate.

ignored since it is not received at receiver i. The capacity region of this three user MAC will have seven constrains including the first four given in (19). The modification to this is that we drop the remaining three constraints by following similar arguments as in [12]. First, recall that a constraint for a MAC region that involves the rates $\{R_i : i \in \mathcal{M}\}$ for some subset of the users $\mathcal{M}$ corresponds to a bound on an error event where the message for each user in $\mathcal{M}$ is in error and all other messages are correct [13]. The missing bounds correspond to error events that we ignore for one of the following two reasons. First, due to the use of superposition coding we can ignore constraints which correspond to making an error in the common message but not the private message. Second, from the point-of-view of user i, we can ignore the constraint that corresponds to an error in only user j's signal.

From the above discussion and following similar arguments as in [3], we then have the following characterization of the rate-tuples that be achieved with this class of randomized Han-Kobayashi schemes.

Lemma 3: $\mathcal{R}_{RHK} = \mathcal{R}_1^m \cap \mathcal{R}_2^m$ is an achievable region for randomized Han-Kobayashi schemes.

If a rate-tuple is in $\mathcal{R}_{RHK}$, receiver i may not be able to reliably decode user j's common and common random messages. In particular, this must be true if $R_{jc} + R_{jr} > n_{ij}$. However, if $R_{ic} + R_{ip} \geq L_i$ for $i = 1, 2$, then the next lemma shows that this will be possible.

Lemma 4: Any rate-tuple $(R_{1c}, R_{1r}, R_{1p}, R_{2c}, R_{2r}, R_{2p})$ in the interior of $\mathcal{R}_{RHK}$ with $R_{ic} + R_{ip} \geq L_i$ for $i = 1, 2$ can be achieved by a randomized Han-Kobayashi scheme in which each user i decodes user j's common and common random message (with arbitrarily small probability of error).

Proof: First note that if a rate tuple is in the interior of $\mathcal{R}_{RHK}$ with $R_{ic} + R_{ip} \geq L_i = (n_{ii} - n_{ij})^+$, then from the first constraint in (19) for receiver i, it follows that

$$R_{jc} + R_{jr} < n_{ij}. \tag{20}$$

Now consider a randomized Han-Kobayashi scheme which achieves this rate tuple. After user i decodes his own private and common messages, he will have a clean view of user j's common message. Moreover, from the above constraint, the rate of this message is less than the capacity of the channel from user j to receiver i and so there must exist a randomized Han-Kobayashi

scheme in which user i can also decode user j's common messages with arbitrary reliability.[9] ■

A given rate tuple $\mathbf{R} = (R_{1c}, R_{1r}, R_{1p}, R_{2c}, R_{2r}, R_{2p})$ is defined to be *self-saturated* at receiver i if $\mathbf{R} \in \mathcal{R}_i^m$ at receiver i, but any other choice of R_{ic}, R_{ir} and R_{ip} with a larger value of $R_{ic}+R_{ip}$ (keeping all other rates fixed) will result in a rate-tuple that is not in $\mathcal{R}_i^m$. Clearly, a self-saturated rate-tuple must lie on the boundary of the modified MAC region at receiver i. Additionally, the constraints in (19) that are tight at this point must involve both R_{ip} and R_{ic}. If a rate-pair is self-saturated and $R_{ip} + R_{ic} \geq L_i$, then it will be useful to think about this in the context of a second MAC region at receiver i in which there are only two users, one corresponding to user i's entire message (at rate $R_i = R_{ic} + R_{ip}$) and a second that again corresponds to the common and common random messages sent by user j (at rate $R_{jc} + R_{jr}$). This MAC region is given by

$$\begin{aligned} R_i + R_{jc} + R_{jr} &\leq \max(n_{ii}, n_{ij}) \\ R_i &\leq n_{ii} \\ R_{jc} + R_{jr} &\leq n_{ij}. \end{aligned} \tag{21}$$

It can seen that if a rate tuple is self-saturated for user i and satisfy $R_{ic} + R_{ip} \geq L_i$, then the rates R_i and $R_{jc} + R_{jr}$ must be in (21) and if R_i is increased by any amount this will no longer be true. We use this to show that if a user is self-saturated and his rate is greater than L_i, then he can not deviate and improve his pay-off. The key idea here is that if a user could improve, then he will violate one of the MAC constraints in (21). This cannot be possible since after deviating and decoding his own message, he should still be able to decode the other user's message. The next lemma formalizes this argument.

Lemma 5: If a rate tuple $(R_{1c}, R_{1r}, R_{1p}, R_{2c}, R_{2r}, R_{2p})$ is self-saturated with $R_{ic} + R_{ip} \geq L_i$ for both both receivers i, then $(R_{1p} + R_{1c}, R_{2p} + R_{2c}) \in \mathcal{C}_{\mathrm{NE}}$.

Proof: Given a rate tuple $\mathbf{R} = (R_{1c}, R_{1r}, R_{1p}, R_{2c}, R_{2r}, R_{2p})$ that is self-saturated at both receivers and with $R_{ic} + R_{ip} \geq L_i$, it follows from Lemmas 3 and 4 that for any $\eta > 0$, and any $\epsilon > 0$, there exists a randomized Han-Kobayashi scheme achieving rates $(R_{1c} - \eta/6, R_{1r} - \eta/6, R_{1p} - \eta/6, R_{2c} - \eta/6, R_{2r} - \eta/6, R_{2p} - \eta/6)$ for which each receiver can decode both his

[9] The Han-Kobayashi scheme under consideration may suffice, however if this scheme does not provide a low enough probability of error, then a new scheme that achieves the same rate-tuple with the desired probability of error can be found (perhaps by using a longer blocklength).

own common and private messages as well as the other users common and common random messages with probability of error less than ϵ.

Next we argue that for ϵ small enough such a pair of strategies must be a η-NE of a ϵ-game. First note that under these nominal strategies each user i will receive a pay-off of $R_{ic}+R_{ip}-\eta/3$. Assume that these strategies are not an equilibrium, and without loss of generality suppose that user 1 can deviate and improve his performance by at least η. After deviating, the rates for the MAC region at user 1 in (21) are given by $\tilde{\mathbf{R}} = (\tilde{R}_1, R_{2c}+R_{2r}-\eta/3)$, where $\tilde{R}_1 \geq R_1 + 2\eta/3$. Since the rate tuple $\mathbf{R}$ is self-saturated, it follows that after this deviation, the rates $\tilde{\mathbf{R}}$ must violate either the first or second constraint in (21) for $i=1$ by at least $\eta/3$.

Suppose that user 1 deviates to a blocklength N_1 strategy. Then using Fano's inequality (as in (13)) for the average bit error probability and following the usual converse for a MAC channel, it must be that

$$\tilde{R}_1 \leq \frac{I(\mathbf{x}_1;\mathbf{y}_1|\mathbf{x}_{2c})}{N_1} + \delta \tag{22}$$

where δ goes to zero as the average bit error probability ϵ does. In particular, choosing ϵ small enough so that δ is less than $\eta/3$, then (22) implies that

$$\tilde{R}_1 < n_{11} + \eta/3.$$

Hence, the second constraint in (21) can not be violated.

Likewise, since in the nominal strategy for user 1, he was able to decode user j's common and common random signals, it follows that

$$R_{2c} + R_{2p} \leq \frac{I(\mathbf{x}_{2c};\mathbf{y}_1)}{N_i'} + \delta' \tag{23}$$

where N_i' denotes the block length used in the nominal strategy. Combining (22) and (23) and choosing ϵ small enough so that $\delta+\delta' < \eta/3$, we have[10]

$$\tilde{R}_1 + R_{2c} + R_{2p} \leq \max(n_{ii}, n_{ij}) + \eta/3 \tag{24}$$

which shows that the first constraint in (21) can not be violated.

[10]Note, as discussed in Section II, we need to replace (22) and (23) with the corresponding expressions over super-blocks whose length is the least common multiple of N_i and N_i' so that that both equations are over the same block-length.

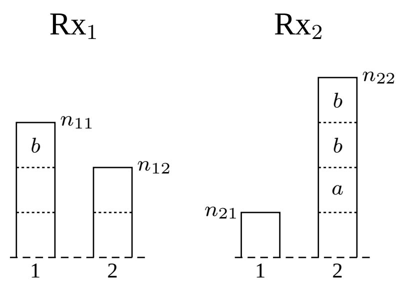

Fig. 7. This figure shows the interference free levels for each transmitter in the interference channel from Fig. 4. The levels for user i are indicated at that user's receiver by either a "a" or a "b". The levels labeled with a "a" correspond to "private levels," which are accounted for in (25). The levels labeled with a "b" correspond to common levels, which are accounted for in (26).

Therefore, such a deviation cannot exist and the nominal strategy must be a η-NE for small enough ϵ. Taking the limit as $\eta \to 0$, it follows that the desired rates must lie in $\mathcal{C}_{\mathrm{NE}}$. ■

To prove that $\mathcal{C}_{\mathrm{NE}} = \mathcal{C} \cap \mathcal{B}$, we will show for that any rate-point $(R_1, R_2) \in \mathcal{C} \cap \mathcal{B}$, there exists a feasible rate tuple in $\mathcal{R}_{RHK}$ with $R_i = R_{ic} + R_{ip}$ for $i = 1, 2$ and that is self-saturated at both receivers. The desired result then follows directly from Lemma 5. As a first step toward doing this, we define a class of non-randomized Han-Kobayashi rates $(R_{1c}, R_{1p}, R_{2c}, R_{2p})$ at which each transmitter is fully utilizing its "interference-free" levels, i.e., the $L_i = (n_{ii} - n_{ij})^+$ most significant levels at transmitter i. Depending on the channel some of these levels may be common and some may be private. Specifically, there are

$$a_i = (n_{ii} - n_{ji} - n_{ij})^+$$

private interference free levels at user i and

$$b_i = (n_{ii} - \max(n_{ii} - n_{ji}, n_{ij}))^+$$

common interference free levels at user i. An example of these is shown in Fig. 7. We say that $(R_{1c}, R_{1p}, R_{2c}, R_{2p})$ *fully utilizes the interference free levels* for user i if

$$R_{ip} \geq a_i \tag{25}$$

$$R_{ic} \geq b_i. \tag{26}$$

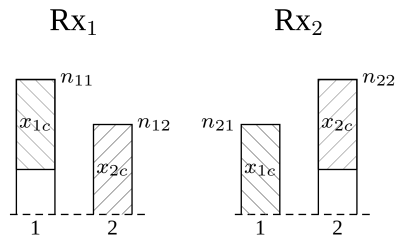

Fig. 8. An example of a non-randomized Han-Kobayashi scheme for a symmetric channel with $n_{ii} = 3$ and $n_{ij} = 2$. Here each user is using the rate split $R_{ic} = 1$ and $R_{ip} = 0$, which fully utilizes the 1 interference free-level at each transmitter. This rate split is not self-saturating at either transmitter.

Lemma 6: If $(R_{1c}, R_{1p}, R_{2c}, R_{2p})$ *fully utilizes the interference free levels* for each user i, then there exists random common rates $R_{1r}, R_{2r} \geq 0$ such that $(R_{1c}, R_{1r}, R_{1p}, R_{2c}, R_{2r}, R_{2p})$ is self-saturated at both receivers.

Proof: The intuition is as follows. If $R_{ip} \geq a_i$ and $R_{ic} \geq b_i$, then all the interference-free levels are saturated at receiver i (i.e., at maximum entropy). The remaining n_{ij} levels are all reachable by the common signal from user j. By putting sufficient number of random bits on that common signal, these n_{ij} levels can be fully saturated as well.

More rigorously, we will show that one can always increase R_{2r} such that the overall sum rate constraint (the first constraint in (19)) is tight, so that receiver 1 is saturated. Suppose no such choice of R_{2r} exists. Then, it must be that R_{2r} cannot be further increased because the second constraint in (19) for receiver i is tight. However, if this constraint is tight, then since $R_{1c} \geq b_i$, it can be seen that the first constraint at receiver i must also be tight. ■

As an example of the construction used in the proof of Lemma 6 consider a symmetric channel with $n_{11} = n_{22} = 3$ and $n_{12} = n_{21} = 2$ as shown in Fig. 8. Each user has 1 interference free level with $a_i = 0$ and $b_i = 1$. Thus the non-randomized Han-Kobayashi rates given by $R_{1c} = R_{2c} = 1$ and $R_{1p} = R_{2p} = 0$ fully utilize the interference free levels at each receiver, but is not self-saturating since, each transmitter could increase R_{ip} by one. However, if each transmitter sets $R_{ir} = 1$, the resulting randomized rates will be self-saturated as shown in Fig. 9

It follows from [3] that for any rate pair in $\mathcal{C} \cap \mathcal{B}$, there exists a non-randomized Han-Kobayashi rate-split that satisfies Lemma 3. If these Han-Kobayashi rates fully utilize the interference-

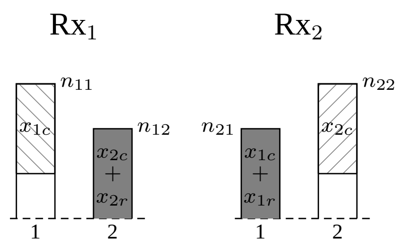

Fig. 9. A randomized Han-Kobayashi scheme which achieves the same rates as the non-randomized scheme in Fig. 9 but is self-saturated. Here we do not show R_{ir} at receiver i since this signal can be removed from the assumed common randomness.

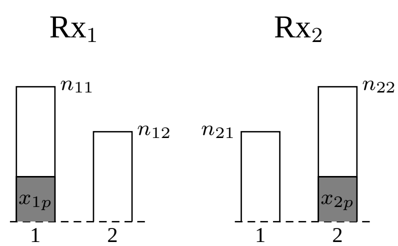

Fig. 10. An example of an alternative non-randomized Han-Kobayashi rate-split that achieves the same rates as the scheme in Fig. 8 but does not fully utilize the interference-free levels.

free levels then we are done. Unfortunately, not all non-randomized Han-Kobayashi rates fully utilize the interference-free levels. For example consider the symmetric channel in the previous paragraph. An alternative non-randomized Han-Kobayashi rate-split is given by $R_{1c} = R_{2c} = 0$ and $R_{1p} = R_{2p} = 1$ (see Fig. 10). These rates do not fully utilize the interference-free levels and cannot be made into an equilibrium by simply increasing the users' common random rates. Though this set of rates do not fully utilize the interference-free levels, as the previous example illustrates, there is another set of non-randomized rates that do. The next lemma generalizes this example.

Lemma 7: Given any point $(R_1, R_2) \in \mathcal{C} \cap \mathcal{B}$, then there exists a non-randomized Han-Kobayashi rate split that fully utilizes the interference free levels at each transmitter.

Proof: To prove this lemma we begin with an arbitrary non-randomized Han-Kobayashi

rate-split that satisfies Lemma 3 and show that this can always be transformed into one that fully utilizes the interference free levels at each transmitter. A key point here is that $a_i + b_i = L_i$, so that for any point in $\mathcal{B}$ there will always be sufficient amount of "rate" available to meet the constraints in (25) and (26).

First we show that if $R_{ip} < a_i$ for either user i, then we can always increase R_{ip} and decrease R_{ic} by the same amount until $R_{ip} = a_i$. The only way such a transformation could not be done is if the second constraint in (19) at receiver i prevented it. But by combining (20) at receiver j and the second constraint at receiver i, it can be seen that this will never happen for $R_{ip} < a_i$.

Thus we can assume that $R_{ip} \geq a_i$. Given this if a rate-pair does not fully utilize the interference free levels at transmitter i, it must be that $R_{ic} < b_i$. Suppose that this is true for receiver 1 and consider increasing R_{1c} and decreasing R_{1p} by the same amount until $R_{1c} = b_1$. Note that since $a_1 + b_1 = L_1$ and $R_{ic} + R_{ip} \geq L_1$, when we decrease R_{1p} in this way it will never cause it to become less than a_1. Changing R_{1p} and R_{1c} in this manner will not violate any of the constraints in (19) at receiver 1, since every constraint involving R_{1c} also involves R_{1p}. If this can be done without violating the first or second constraints at receiver 2 then we are done. Otherwise, it must be that at least one of these constraints becomes tight when R_{1c} reaches the value $R_{1c}^* = b_1 - \Delta$, for some $\Delta > 0$. Note that

$$\begin{aligned} R_{2p} + R_{2c} + R_{1c}^* &= R_2 + R_{1c}^* \\ &\leq U_2 + b_1 - \Delta \qquad (27) \\ &= \max(n_{22}, n_{21}) - \Delta \end{aligned}$$

and so the first constraint at receiver 2 can not be tight. This implies that the second constraint at receiver 2 must be tight, i.e.,

$$R_{2p} + R_{1c}^* = \max(n_{21}, n_{22} - n_{12}). \qquad (28)$$

Combining this with (27) we have

$$\begin{aligned} R_{2c} &\leq \max(n_{22}, n_{21}) - \max(n_{21}, n_{22} - n_{12}) - \Delta \\ &= b_2 - \Delta. \end{aligned}$$

It then follows that $R_{2p} \geq a_2 + \Delta$. In other words user 2's interference free levels must be "underutilized" by at least as much as user 1's.

Now consider increasing R_{1c} from R_{1c}^* by Δ while simultaneously reducing R_{1p} and R_{2p} each by Δ and also increasing R_{2c} by Δ. The above calculations show that no constraint will be violated if we only changed R_{1c}, R_{1p} and R_{2p} in this way. Likewise, by applying the same argument to R_{2c}, R_{2p} and R_{1p}, changing these values will not violate any constraints. The only possible violation could occur in the first constraints at either MAC, which involves both R_{1c} and R_{2c}. However this constraint also involves one of the users' private rates, and so cannot be violated by the same argument as in (27). After this transformation, the resulting rate-split will fully utilize the interference free levels at both receivers. ■

Combining Lemmas 5, 6 and 7, we have proven Theorem 1.

We also note that by direct calculation it can be shown that $\mathcal{C}_{NE}$ always contains at least one efficient point, i.e., one point that is sum-rate optimal. Indeed it can be shown that for a symmetric channel, for $\alpha \leq 2/3$, the only efficient point in $\mathcal{C}_{\mathrm{NE}}$ is the symmetric sum-rate optimal point, while for $\alpha \geq 2/3$ all sum-rate optimal points are in $\mathcal{C}_{\mathrm{NE}}$.

IV. The Gaussian IC

In the previous section, we completely characterized the Nash equilibrium region for the two-user linear-deterministic IC. In this section, we show that an analogous result holds for the Gaussian channel model within a one bit approximation.

A. Gaussian Channel Model

Here, our goal is to characterize rates in $\mathcal{C}_{\mathrm{NE}}$ for two-user Gaussian interference channels represented by (see Fig. 1)

$$\begin{aligned} y_1 &= h_{11}x_1 + h_{12}x_2 + z_1 \\ y_2 &= h_{21}x_1 + h_{22}x_2 + z_2 \end{aligned} \tag{29}$$

where for $i = 1, 2$, $z_i \sim \mathcal{CN}(0, 1)$ and the input $x_i \in \mathbb{C}$ is subject to the power constraint $\mathrm{E}[|x_i|^2] \leq P$. Following [1], for $i = 1, 2$, we parameterize this channel by the signal-to-noise ratios $\mathsf{SNR}_i = P|h_{ii}|^2$ and the interference-to-noise ratios $\mathsf{INR}_{ij} = P|h_{ij}|^2$.

The characterization of $\mathcal{C}_{\mathrm{NE}}$ in the linear deterministic case relied on knowing the exact capacity region $\mathcal{C}$ for the deterministic IC and that any point in this region can be exactly achieved by a non-randomized Han-Kobayashi schemes. For the Gaussian IC, $\mathcal{C}$ is only known in the case

of very weak [15]–[17] or very strong interference [2], [18]. Otherwise, $\mathcal{C}$ is not known exactly but in [1] it is characterized to "within one bit" for all parameter ranges. Furthermore, [1] shows that in a general Gaussian IC, we can achieve any point within one bit by a non-randomized Han-Kobayashi scheme. These one-bit gaps will effect how accurately we can characterize $\mathcal{C}_{\rm NE}$ in the Gaussian case. Namely, in general we will also be able to characterize this region only to within a one bit gap (though for particular channels this gap may be smaller).

B. Main Results

For the Gaussian IC our main result is to show an analogue to Theorem 1 that characterizes $\mathcal{C}_{\rm NE}$ to within one bit. In this case we will give both an inner bound and outer bound on $\mathcal{C}_{\rm NE}$. Both of these bounds will be given in terms of a capacity region and a "box" as in the deterministic case. The true capacity region will be used for the outer bound, while an achievable "Han-Kobayashi" region, $\mathcal{C}_{HK}$, will be used for the inner bound. Here, $\mathcal{C}_{HK}$ corresponds to the set of rates that are achievable using the specific class of Han-Kobayashi schemes in [1] (this will be defined more precisely in Sect. IV-C2). This region is within 1-bit of the capacity region $\mathcal{C}$, i.e. if $(R_1, R_2) \in \mathcal{C}$, then $((R_1 - 1)^+, (R_2 - 1)^+ \in \mathcal{C}_{HK}$.

The box $\mathcal{B}$ used for the outer bound is given by

$$\mathcal{B} = \{(R_1, R_2) : L_i \leq R_i \leq U_i, \forall i = 1, 2\},$$

where for each user $i = 1, 2$

$$L_i := \log\left(1 + \frac{\mathsf{SNR}_i}{1 + \mathsf{INR}_i}\right),$$

and

$$U_i = \min\left\{\log(1 + \mathsf{SNR}_i + \mathsf{INR}_{ij}) - \log\left(1 + \frac{[\mathsf{SNR}_j - \max(\mathsf{INR}_{ji}, \mathsf{SNR}_j/\mathsf{INR}_{ij})]^+}{1 + \mathsf{INR}_{ji} + \max(\mathsf{INR}_{ji}, \mathsf{SNR}_j/\mathsf{INR}_{ij})}\right),\right.$$
$$\left.\log(1 + \mathsf{SNR}_i)\right\}. \tag{30}$$

While the inner bound is given in terms of the "box,"

$$\mathcal{B}^- = \{(R_1, R_2) : L_i \leq R_i \leq \max(U_i - 1, L_i), \forall i = 1, 2\},$$

which differs from $\mathcal{B}$ by at most one bit.

We next state the analogous result to Theorem 1.

Theorem 2: $\mathcal{C}_{HK} \cap B^- \subseteq \mathcal{C}_{\rm NE} \subseteq \mathcal{C} \cap \mathcal{B}$. Moreover, for a Gaussian IC with strong interference, $\mathcal{C}_{\rm NE} = \mathcal{C}_{HK} \cap B$.

In certain cases, when we know additional properties of the capacity region we can strengthen these results. For example for very weak interference, from the results in [15]–[17] it is known that the maximum sum-rate in $\mathcal{C}$ is achieved by simply treating interference as noise, which is also in $\mathcal{C}_{HK}$. This corresponds exactly to the lower-left corner of $\mathcal{B}$ and $\mathcal{B}^-$. Hence, Theorem 2 implies that $\mathcal{C}_{\rm NE}$ contains the single point (L_1, L_2) and thus in this case $\mathcal{C}_{\rm NE}$ is characterized exactly.

We also note that the bounds in Theorem 2 can be shown to be within a constant gap of the bounds given in Theorem 1 for a related deterministic IC, which is obtained by the mapping $n_{ii} = \lfloor \log(\mathsf{SNR}_i) \rfloor$ and $n_{ij} = \lfloor \log(\mathsf{INR}_{ij}) \rfloor$.

In the next section we will give a proof of Theorem 2 that is based on generalizing each of the steps we used in the deterministic case.

C. Proofs

1) Non-equilibrium points: We begin by showing that certain rate-pairs can not be in $\mathcal{C}_{\rm NE}$.

Lemma 8: If $(R_1, R_2) \in \mathcal{C}_{\rm NE}$, then $R_i \geq L_i := \log(1 + \frac{\mathsf{SNR}_i}{1+\mathsf{INR}_i})$ for $i = 1, 2$.

Proof: Regardless of user j's strategy, user i can always achieve at least rate $\log(1+\frac{\mathsf{SNR}_i}{1+\mathsf{INR}_i})$ (with arbitrarily small probability of error) by treating user j's signal as noise. Hence, this is always a possible deviation for user i in any ϵ-game. Thus user i's rate in any η-NE must be at least $L_i - \eta$. ■

The bound in Lemma 8 is a direct analog to the bound in Lemma 1 for the linear deterministic channel, which characterizes the lower bounds of the box $\mathcal{B}$. The next lemma gives an upper bound corresponding to the bound in Lemma 2.

Lemma 9: If $(R_1, R_2) \in \mathcal{C}_{\rm NE}$, then $R_i \leq U_i$, where U_i is given in (30).

Proof: Suppose $(R_1, R_2) \in \mathcal{C}_{\rm NE}$. Without loss of generality, let us focus on user 1 to show the upper bound on R_1.

We define first a parameter:

$$\sigma_v^2 := \max\left(\frac{\mathsf{INR}_{21}}{\mathsf{SNR}_2}, \frac{1}{\mathsf{INR}_{12}}\right). \tag{31}$$

Consider first the case that $\sigma_v^2 > 1$: this corresponds to the case in the deterministic channel when the interference from the signal user 2 transmits at its interference-free levels appears

below noise level at receiver 1. In this case, there will be no minimum amount of interference that user 2 will cause to user 1 at a NE, and we simply bound R_1 by its point-to-point capacity:

$$R_1 \leq \log(1 + \mathsf{SNR}_1). \tag{32}$$

The case when $\sigma_v \leq 1$ is more interesting and we need a tighter bound on R_1. Fix $\eta > 0$ and arbitrary. Given a sufficiently small $\epsilon > 0$, there exist a $(1-\epsilon)$-reliable strategy pair (s_1^*, s_2^*) achieving the rate pair (R_1, R_2) that is also a η-NE. As remarked in Section II, we can assume that in this nominal strategy pair, both users use a common block length N. Applying Fano's inequality to user 1 for average bit error probability, we get the bound, for any block k:

$$R_1 \leq \frac{I(m_1; \mathbf{y}_1 | \omega_1)}{N} + \delta \tag{33}$$

where $\mathbf{y}_1$ is user 1's received signal over the block, δ depends on ϵ and goes to zero as ϵ goes to zero, and ω_1 denotes any common randomness in $\mathbf{x}_1$. Note that we drop the block indices of the message and the signals to simplify notation. Now,

$$\begin{aligned}
&\frac{1}{N} I(m_1; \mathbf{y}_1 | \omega_1) \\
\leq\;& \frac{1}{N} I(\mathbf{x}_1; \mathbf{y}_1 | \omega_1) \\
\leq\;& \frac{1}{N} I(\mathbf{x}_1; \mathbf{y}_1) \\
=\;& \frac{1}{N} \left[h(\mathbf{y}_1) - h(\mathbf{y}_1 | \mathbf{x}_1) \right] \\
=\;& \frac{1}{N} \left[h(\mathbf{y}_1) - h(\mathbf{z}_1) - h(\mathbf{y}_1 | \mathbf{x}_1) + h(\mathbf{z}_1) \right] \\
\leq\;& \log(1 + \mathsf{SNR}_1 + \mathsf{INR}_{12}) - \frac{I(\mathbf{x}_2; \tilde{\mathbf{y}}_1)}{N}
\end{aligned}$$

where

$$\tilde{\mathbf{y}}_1 := h_{12} \mathbf{x}_2 + \mathbf{z}_1.$$

Combining this with the above inequality, we get:

$$R_1 \leq \log(1 + \mathsf{SNR}_1 + \mathsf{INR}_{12}) - \frac{I(\mathbf{x}_2; \tilde{\mathbf{y}}_1)}{N}. \tag{34}$$

The term $I(\mathbf{x}_2;\tilde{\mathbf{y}}_1)$ plays the role of $H(\mathbf{s}_2)$ in the linear deterministic case. We now seek a lower bound on $I(\mathbf{x}_2;\tilde{\mathbf{y}}_1)$. Applying Fano's inequality to user 2, we get

$$\begin{aligned} R_2 &\leq \frac{1}{N} I(m_2;\mathbf{y}_2|\omega_2) + \delta \\ &\leq \frac{1}{N} I(\mathbf{x}_2;\mathbf{y}_2) + \delta \\ &= \frac{1}{N} I(\mathbf{u}_2,\mathbf{v}_2;\mathbf{y}_2) + \delta \end{aligned}$$

where

$$\mathbf{u}_2 = \mathbf{x}_2 + \mathbf{v}_2$$

and $\mathbf{v}_2 \sim \mathcal{CN}(0, \sigma_v^2 I_N)$ independent of everything else, with σ_v^2 defined as in (31).

Now,

$$\begin{aligned} I(\mathbf{u}_2,\mathbf{v}_2;\mathbf{y}_2) &= I(\mathbf{u}_2;\mathbf{y}_2) + I(\mathbf{v}_2;\mathbf{y}_2|\mathbf{u}_2) \\ &= I(\mathbf{u}_2;\mathbf{y}_2) + I(\mathbf{v}_2;\tilde{\mathbf{y}}_2|\mathbf{u}_2) \\ &\leq I(\mathbf{u}_2;\mathbf{y}_2) + I(\mathbf{v}_2;\tilde{\mathbf{y}}_2) \end{aligned}$$

where

$$\tilde{\mathbf{y}}_2 = \mathbf{y}_2 - h_{22}\mathbf{u}_2 = h_{22}\mathbf{v}_2 + h_{21}\mathbf{x}_1 + \mathbf{z}_2$$

and the last inequality above follows from the Markov chain $\mathbf{u}_2 - \mathbf{v}_2 - \tilde{\mathbf{y}}_2$.

Combining this with the previous equation, we get:

$$R_2 \leq \frac{1}{N} I(\mathbf{u}_2;\mathbf{y}_2) + \frac{1}{N} I(\mathbf{v}_2;\tilde{\mathbf{y}}_2) + \delta. \tag{35}$$

Let us now consider an alternative strategy s_2' for user 2: a superposition of two i.i.d. Gaussian codebooks, one with each component of each codeword having variance $1-\sigma_v^2$, and one with each component of each codeword having variance σ_v^2. The codes have block length NK. If we choose $K \to \infty$, then standard random coding argument and the chain rule of mutual information implies that this scheme can achieve a rate of:

$$\frac{1}{N} I(\tilde{\mathbf{u}}_2,\mathbf{v}_2;\mathbf{y}_2') = \frac{1}{N} I(\tilde{\mathbf{u}}_2;\mathbf{y}_2') + \frac{1}{N} I(\mathbf{v}_2;\tilde{\mathbf{y}}_2),$$

where $\tilde{\mathbf{u}}_2 \sim \mathcal{CN}(0, 1-\sigma_v^2 I_N)$ and $\tilde{\mathbf{u}}_2$ and $\mathbf{v}_2$ are independent, and

$$\mathbf{y}_2' = h_{22}(\tilde{\mathbf{u}}_2 + \mathbf{v}_2) + h_{21}\mathbf{x}_1 + \mathbf{z}_2.$$

We have:

$$\frac{1}{N}I(\tilde{\mathbf{u}}_2;\mathbf{y}_2') \geq \log\left(1+\frac{\mathsf{SNR}_2(1-\sigma_v^2)}{1+\mathsf{SNR}_2\sigma_v^2+\mathsf{INR}_{21}}\right),$$

using a worst-case Gaussian noise argument [20] on $\mathbf{v}_2$ and $\mathbf{x}_1$. This implies from (35) that

$$\frac{1}{N}I(\mathbf{u}_2;\mathbf{y}_2) \geq \log\left(1+\frac{\mathsf{SNR}_2(1-\sigma_v^2)}{1+\mathsf{SNR}_2\sigma_v^2+\mathsf{INR}_{21}}\right)-\eta \tag{36}$$

by definition that we are operating at a η-NE.

Next we relate $I(\mathbf{x}_2;\tilde{\mathbf{y}}_1)$ to $I(\mathbf{u}_2;\mathbf{y}_2)$ and complete the argument.

$$\begin{aligned}
& I(\mathbf{x}_2;\tilde{\mathbf{y}}_1) \\
= \;& I(\mathbf{x}_2;h_{12}\mathbf{x}_2+\mathbf{z}_1) \\
= \;& I(\mathbf{x}_2;\mathbf{x}_2+\tilde{\mathbf{z}}_1), \qquad \tilde{\mathbf{z}}_1 \sim \mathcal{CN}(0,\frac{1}{\sqrt{\mathsf{INR}_{12}}}I_N) \\
\geq \;& I(\mathbf{x}_2;\mathbf{x}_2+\mathbf{v}_2), \qquad \text{since } \sigma_v^2=\max(\tfrac{\mathsf{INR}_{21}}{\mathsf{SNR}_2},\tfrac{1}{\mathsf{INR}_{12}}) \geq \tfrac{1}{\mathsf{INR}_{12}} \\
= \;& I(\mathbf{u}_2,\mathbf{x}_2) \\
\geq \;& I(\mathbf{u}_2;\mathbf{y}_2) \\
\geq \;& N\left[\log\left(1+\frac{\mathsf{SNR}_2(1-\sigma_v^2)}{1+\mathsf{SNR}_2\sigma_v^2+\mathsf{INR}_{21}}\right)-\eta\right] \qquad \text{from (36).}
\end{aligned}$$

Substituting this into (34), we get the final result:

$$\begin{aligned}
R_1 &\leq \log(1+\mathsf{SNR}_1+\mathsf{INR}_{12})-\log\left(1+\frac{\mathsf{SNR}_2(1-\sigma_v^2)}{1+\mathsf{SNR}_2\sigma_v^2+\mathsf{INR}_{21}}\right)+\eta \\
&= \log(1+\mathsf{SNR}_1+\mathsf{INR}_{12})-\log\left(1+\frac{\mathsf{SNR}_2-\max(\mathsf{INR}_{21},\mathsf{SNR}_2/\mathsf{INR}_{12})}{1+\mathsf{INR}_{21}+\max(\mathsf{INR}_{21},\mathsf{SNR}_2/\mathsf{INR}_{12})}\right)+\eta.
\end{aligned}$$

Combining this with inequality (32) and letting $\eta \to 0$ yields the desired result. ■

2) Achievable Nash Equilibrium: The lemmas in the previous section provide an outer bound on $\mathcal{C}_{\mathrm{NE}}$. In this section we give an inner bound on $\mathcal{C}_{\mathrm{NE}}$ by showing that this set contains $\mathcal{C}_{HK}\cap\mathcal{B}^-$. Motivated by the deterministic analysis, we again consider a modified Han-Kobayashi scheme in which each user may send a private message, a common message, and also a common random message that is generated using the common randomness they share with their own receiver. Additionally, in the Gaussian case, we allow a user to also send a private random message using their common randomness. In the deterministic case, sending such a message would not serve any purpose since a user's private signal does not appear at all at the other receiver. In the Gaussian case, a user's private signal is present at the other user and the private random

message is used to ensure that the effect of this signal is essentially the same as "noise." All of these messages are again encoded using a superposition code, which we define formally next.

For a given Gaussian IC, let P_{ip} and P_{ic} denote a user's private and common power respectively, where $P_{ip} + P_{ic} = P$. As in [1], we assume that P_{ip} is set as follows:

$$|h_{ji}|^2 P_{ip} = \begin{cases} \min(1, \mathsf{INR}_{ji}), & \text{if } \mathsf{INR}_{ji} < \mathsf{SNR}_j \\ 0, & \text{otherwise.} \end{cases}$$

Let $\mathsf{INR}_{ji}^p = |h_{ji}|^2 P_{ip}$ denote the INR at receiver j due to this choice of P_{ip} and let $\mathsf{SNR}_i^p = |h_{ii}|^2 P_{ip}$ denote the corresponding SNR at receiver i. Note that when $\mathsf{SNR}_j > \mathsf{INR}_{ji} > 1$, the received interference power at receiver j due to user i's private power is at the noise level.

As in the deterministic case, we define a randomized Han-Kobayashi scheme to be one in which each user i separates their message set into $\mathcal{M}_{ip} \times \mathcal{M}_{ic}$ with rates R_{ip} and R_{ic}, respectively, and also generates a random common message set Ω_{ir} with rate R_{ir}. Additionally, we allow each transmitter to generate a random private message set, Ω_{is} with rate R_{is}. These messages are then encoded using a superposition code as follows. First, the transmitter encodes the common message $m_{ic} \in \mathcal{M}_{ic}$ and common random message $\omega_{ir} \in \Omega_{ir}$ into a codeword $\mathbf{x}_{ic}(m_{ic}, \omega_{ir})$ from a codebook that satisfies the average power constraint of P_{ic}. Given this codeword, it encodes the private message m_{ip} and private random message $\omega_{is} \in \Omega_{is}$ into a codeword $\mathbf{x}_{ip}(m_{ip}, \omega_{is}, \mathbf{x}_{ic})$ from a codebook that is indexed by the common codeword $\mathbf{x}_{ic}$ and satisfies the average power constraint of P_{ip}. It then transmits the superposition $\mathbf{x}_i = \mathbf{x}_{ic} + \mathbf{x}_{ip}$. As in the deterministic case, we call such a scheme $(1-\epsilon)$-reliable if a user is able to decode both his common and private messages with reliability of $(1-\epsilon)$.

We again introduce a modified MAC region $\mathcal{R}_i^m$ for each receiver i that we will use to characterize the rates achievable by such a scheme. This is the set of rate tuples that satisfy:

$$\begin{aligned} R_{ic} + R_{ip} + R_{jc} + R_{jr} &\le \log\left(1 + \frac{\mathsf{SNR}_i + \mathsf{INR}_{ij} - \mathsf{INR}_{ij}^p}{\mathsf{INR}_{ij}^p + 1}\right) \\ R_{ip} + R_{jc} + R_{jr} &\le \log\left(1 + \frac{\mathsf{SNR}_i^p + \mathsf{INR}_{ij} - \mathsf{INR}_{ij}^p}{\mathsf{INR}_{ij}^p + 1}\right) \\ R_{ip} &\le \log\left(1 + \frac{\mathsf{SNR}_i^p}{\mathsf{INR}_{ij}^p + 1}\right) \\ R_{ic} + R_{ip} &\le \log\left(1 + \frac{\mathsf{SNR}_i}{\mathsf{INR}_{ij}^p + 1}\right). \end{aligned} \tag{37}$$

As in the deterministic case, these constraints arise from considering the three user MAC region at receiver 1 corresponding to the rates R_{ic}, R_{ip} and $R_{jc}+R_{jr}$. Using these regions we then have the following characterization of rate-splits that can achieved with this class of schemes.

Lemma 10: $\mathcal{R}_{RHK} = \mathcal{R}_1^m \cap \mathcal{R}_2^m$ is an achievable region for randomized Han-Kobayashi schemes.

We define the Han-Kobayashi region $\mathcal{C}_{HK}$ in Theorem 2 as the set of rates (R_1, R_2) for which there exists a rate split $(R_{1c}, R_{1r}, R_{1p}, R_{1s}, R_{2c}, R_{2r}, R_{2p}, R_{1s}) \in \mathcal{R}_{RHK}$ with $R_1 = R_{1c} + R_{1p}$ and $R_2 = R_{2c} + R_{2p}$. Using [1], it follows that $\mathcal{C}_{HK}$ is within one-bit of $\mathcal{C}$.[11] Next we give an analog to Lemma 4.

Lemma 11: Any rate-tuple $(R_{1c}, R_{1r}, R_{1p}, R_{1s}, R_{2c}, R_{2r}, R_{2p}, R_{2s})$ in the interior of $\mathcal{R}_{RHK}$ with $R_{ic} + R_{ip} \geq L_i$ for $i = 1, 2$ can be achieved by a randomized Han-Kobayashi scheme in which each user i decodes user j's common and common random message (with arbitrarily small probability of error).

The proof here follows exactly the same steps as in the deterministic case. In particular for $R_{ic} + R_{ip} \geq L_i$, note that

$$R_{jc} + R_{jr} < \log\left(1 + \frac{\mathsf{INR}_{ij} - \mathsf{INR}_{ij}^p}{1 + \mathsf{INR}_{ij}^p}\right) \tag{38}$$

which implies that if user i can decode his own common and private messages, he will be receiving user j's common messages at a rate less than the capacity of the channel over which these messages are sent.

Note also that the private random rate R_{is} does not show up in any of the constraints for $\mathcal{R}_{RHK}$. This is because in these constraints each user i is treating the other user's private message as worst-case noise with power INR_{ij}^p and can remove their own random private message. The next lemma shows that the random private rate can always be chosen so that there is essentially no loss in this assumption.

Lemma 12: Given any $\delta > 0$ and any rate tuple in $\mathcal{R}_{RHK}$ with

$$R_{js} = (\log(1 + \mathsf{INR}_{ij}^p) - R_{jp} - \delta/2)^+$$

[11]The rate region studied in [1] corresponds to the rates R_1 and R_2 that can be achieved with rate-splits in $\mathcal{R}_{RHK}$ where the common random and private random rates of both users are zero. It can be seen that the resulting region is equivalent to $\mathcal{C}_{HK}$ as defined here.

then for a large enough block-length N, this rate tuple can be achieved by a randomized Han-Kobayashi scheme such that

$$\frac{1}{N}h(h_{ij}\mathbf{x}_j^p + \mathbf{z}_1) \geq \log(\pi e(1 + \mathsf{INR}_{ij}^p)) - \delta.$$

Proof: Given a rate tuple that satisfies the conditions of this lemma and a constant $\delta > 0$, we next describe a specific encoding of user j's private messages to ensure the conditions for this lemma hold true. Let $w_j = (m_{jp}, \omega_{js})$ denote the total private message to be encoded by user j (chosen from a total private codebook with rate $R_{jp} + R_{js}$). To encode this message[12] we consider the following two cases: (i) $R_{jp} \leq \log(1 + \mathsf{INR}_{ij}^p) - \delta/2$ and (ii) $R_{jp} > \log(1 + \mathsf{INR}_{ij}^p)$. In each case we will separate w_j into two messages so that $w_j = (w_j^1, w_j^2)$.

Case 1: $R_{jp} \leq \log(1 + \mathsf{INR}_{ij}^p) - \delta/2$. In this case we set $w_j^1 = m_{jp}$, i.e. this is the private message which is to be decoded at receiver j. Since the rate-tuple is in $\mathcal{R}_{RHK}$, this message must be decodable over a Gaussian channel with a capacity of $\log(1 + \mathsf{SNR}_j^p)$. We then set $w_j^= \omega_{js}$, i.e., this is the private random message sent by user j. By assumption this message will have a rate of $R_{js} = \log(1 + \mathsf{INR}_{ij}^p) - R_{jp} - \delta/2$.

By choosing N large enough, there will exist a Gaussian broadcast codebook for these messages so that for a given reliability, w_j^1 and w_j^2 can be received reliably over the Gaussian channel given by

$$y = h_{ij}x_j^p + z_i$$

where x_j^p has average power P_{jp} and the noise variance is 1, and w_j^1 can be received at user j's receiver given w_j^2 (equivalently given the private random message ω_{js}). Note that the first channel has a capacity of $\log(1 + \mathsf{INR}_{ij}^p)$.

By applying Fano's inequality to the first receiver, we have that for a large enough reliability we can find a block-length N so that

$$N(\log(1 + \mathsf{INR}_{ij}^p) - \delta/2) \leq I(w_j; \mathbf{y}) + N\delta/2,$$

where $\mathbf{y} = h_{ij}\mathbf{x}_j^p + \mathbf{z}_1$ denotes the received signal over a block of length N.

[12]To keep the overall superposition code structure for our class of Han-Kobayashi schemes, we need to construct such a private codebook for each codeword $\mathbf{x}_{jc}$ in user j's common codebook. Here we focus on one such codebook.

Now,

$$I(w_j; \mathbf{y}) = h(\mathbf{y}) - N \log(\pi e).$$

Hence, we have

$$h(\mathbf{y}) \geq N(\log(\pi e(1 + \mathsf{INR}_{ij}^p)) - \delta)$$

as desired.

Case 2: $R_{jp} > \log(1 + \mathsf{INR}_{ij}^p)$. In this case we set $R_{js} = 0$ and choose w_j^1 and w_j^2 so that $m_{jp} = (w_j^1, w_j^2)$ where w_j^1 is chosen from a code book with rate $\log(1 + \mathsf{INR}_{ij}^p) - \delta/2$ and w_j^2 is chosen from a codebook with rate $R_{jp} - \log(1 + \mathsf{INR}_{ij}^p) + \delta/2$.

By choosing N large enough, there will exist a Gaussian broadcast codebook for these messages so that for a given reliability, w_j^1 and w_j^2 can be received at user j's receiver, and given w_j^2, w_j^1 can be received reliably over the Gaussian channel

$$y = h_{ij} x_j^p + z_i.$$

Applying Fano's inequality at the second receiver we have

$$NR_{jp} \leq I(w_j; \mathbf{y}, w_j^2) + N\delta/2.$$

Now,

$$\begin{aligned} I(w_j; \mathbf{y}, w_j^2) &= I(w; w_j^2) + I(w; \mathbf{y}|w_j^2) \\ &= N(R_{jp} - \log(1 + \mathsf{INR}_{ij}^p) + \delta/2) \\ &\quad + h(\mathbf{y}|w_j^2) - N\log(\pi e). \end{aligned}$$

Hence, we have

$$h(\mathbf{y}|wj^2) \geq N(\log(\pi e(1 + \mathsf{INR}_{ij}^p)) - \delta).$$

Dropping the conditioning and dividing by N, it follows that

$$\frac{1}{N} h(\mathbf{y}) \geq \log(\pi e(1 + \mathsf{INR}_{ij}^p) - \delta$$

as desired.

■

Note that under the given power constraints, $N\log(\pi e(1 + \mathsf{INR}_{ij}^p))$ is the maximum possible value for $h(\mathbf{y})$ which is achieved when $\mathbf{y}$ is a sequence of i.i.d. Gaussian random variables.

Hence, this lemma can be viewed as showing that when the private rates are sufficiently high, $h(\mathbf{y})$ is well approximated by simply viewing $\mathbf{y}$ as i.i.d. Gaussian.

We say that a rate-split $\mathbf{R}$ is *self saturated* at receiver i if $\mathbf{R} \in \mathcal{R}_i^m$ and any other choice of R_{ic} and R_{ip} in which $R_{ip} + R_{ic}$ is increased (keeping all other rates fixed) will result in a rate-split that is not in $\mathcal{R}_i^m$. Similar to the deterministic case, it can be shown that if receiver i is self-saturated and $R_{ic} + R_{ip} \geq L_i$ then $R_i = R_{ip} + R_{ic}$ and $R_{jc} + R_{jr}$ must be inside the following two user MAC region:

$$\begin{aligned} R_i + R_{jc} + R_{jr} &\leq \log\left(1 + \frac{\mathsf{SNR}_i + \mathsf{INR}_{ij} - \mathsf{INR}_{ij}^p}{\mathsf{INR}_{ij}^p + 1}\right) \\ R_i &\leq \log\left(1 + \frac{\mathsf{SNR}_i}{\mathsf{INR}_{ij}^p + 1}\right) \\ R_{jc} + R_{jr} &\leq \log\left(1 + \frac{\mathsf{INR}_{ij} - \mathsf{INR}_{ij}^p}{1 + \mathsf{INR}_{ij}^p}\right). \end{aligned} \tag{39}$$

Moreover, if R_i is increased then this rate pair will no longer be in this region. Using this we have the next lemma which gives an analogous result to Lemma 5 for the deterministic channel.

Lemma 13: If there exists a rate tuple $\mathbf{R}$ that is self-saturated with $R_{ic} + R_{ip} \geq L_i$ for both both receivers i, then $(R_{1p} + R_{1c}, R_{2p} + R_{2c}) \in \mathcal{C}_{\mathrm{NE}}$.

Proof: The proof follows a similar argument as that for Lemma 5. Given a rate-tuple $\mathbf{R} = (R_{1c}, R_{1r}, R_{1p}, R_{1s}, R_{2c}, R_{2r}, R_{2p}, R_{2s})$ that satisfies the conditions in the lemma, then it follows from Lemmas 10, 11, and 12 that for any $\eta > 0$ and $\epsilon > 0$, there exists a randomized Han-Kobayashi scheme achieving rates $(R_{1c} - \eta/6, R_{1r} - \eta/6, R_{1p} - \eta/6, \tilde{R}_{1s}, R_{2c} - \eta/6, R_{2r} - \eta/6, R_{2p} - \eta/6, \tilde{R}_{2s})$ for which each receiver can decode his own common and private messages as well as the other user's common and common random messages with probability of error less than ϵ. Moreover, by possibly changing the private random rates $\tilde{R}_{1s}$ and $\tilde{R}_{2s}$ to satisfy Lemma 12 such a scheme can be found for which

$$\frac{1}{N} h(h_{ij}\mathbf{x}_j^p + \mathbf{z}_1) \geq \log(\pi e(1 + \mathsf{INR}_{ij}^p)) - \eta/6, \tag{40}$$

for each user i.

Next we argue that for ϵ small enough such a pair of strategies must be a η-NE of a ϵ-game. Assume that these strategies are not an η-NE, and without loss of generality suppose that user 1 can deviate and improve his performance by at least η. After deviating, the rates for the MAC region at user 1 in (39) are given by $\tilde{\mathbf{R}} = (\tilde{R}_1, R_{2c} + R_{2r} - \eta/3)$, where $\tilde{R}_1 \geq R_1 + 2\eta/3$. After

this deviation, the rates $\tilde{\mathbf{R}}$ must violate either the first or second constraint in (39) for $i = 1$ by at least $\eta/3$.

Suppose that user 1 deviates to a blocklength N strategy, which without loss of generality we can assume is the same as the original strategy. Then from Fano's inequality for the average bit error probability it must be that

$$\tilde{R}_1 \leq \frac{I(\mathbf{x}_1; \mathbf{y}_1 | \mathbf{x}_{2c})}{N} + \delta \tag{41}$$

where δ goes to zero as the average bit error probability ϵ does. Note that

$$\begin{aligned}\frac{I(\mathbf{x}_1; \mathbf{y}_1 | \mathbf{x}_{2c})}{N} &= \frac{1}{N}(h(\mathbf{y}_1 | \mathbf{x}_{2c}) - h(\mathbf{y}_1 | \mathbf{x}_1, \mathbf{x}_{2c}) \\ &\leq \log(\pi e(1 + \mathsf{SNR}_1 + \mathsf{INR}_{12}^p) - \log(\pi e(1 + \mathsf{INR}_{12}^p) + \eta/6 \\ &= \log\left(1 + \frac{\mathsf{SNR}_1}{1 + \mathsf{INR}_{12}^p}\right) + \eta/6,\end{aligned}$$

where the second line follows from (40). Choosing ϵ small enough and combining this with (41) implies that

$$\tilde{R}_1 < \log\left(1 + \frac{\mathsf{SNR}_1}{1 + \mathsf{INR}_{12}^p}\right) + \eta/3.$$

Hence, the second constraint in (21) can not be violated.

Likewise, since in the nominal strategy user 1 was able to decode user j's common and common random signals, it follows that

$$R_{2c} + R_{2p} \leq \frac{I(\mathbf{x}_{2c}; \mathbf{y}_1)}{N} + \delta'. \tag{42}$$

Combining (41) and (42) and choosing ϵ small enough so that $\delta + \delta' < \eta/6$, we have

$$\begin{aligned}\tilde{R}_1 + R_{2c} + R_{2p} &\leq \frac{1}{N} I(\mathbf{x}_{2c}, \mathbf{x}_1; \mathbf{y}_1) + \eta/6 \\ &\leq \log(\pi e(1 + \mathsf{SNR}_1 + \mathsf{INR}_{12}) - \log(\pi e(1 + \mathsf{INR}_{12}^p) + \eta/3 \\ &= \log\left(1 + \frac{\mathsf{SNR}_1 + \mathsf{INR}_{12} + \mathsf{INR}_{12}^p}{1 + \mathsf{INR}_{12}^p}\right) + \eta/3\end{aligned}$$

which shows that the first constraint in (21) can not be violated.

Therefore, such a deviation can not exist and the nominal strategy must be a η-NE for small enough ϵ. Taking the limit as $\eta \to 0$, it follows that the desired rates must lie in $\mathcal{C}_{\mathrm{NE}}$. ■

Next, we turn to proving an analogue of Lemma 6 for the Gaussian model. To do this we need to define a parallel notion to the interference-free levels in the deterministic channel. In

a Gaussian channel, this again corresponds to the rate L_i, which can be achieved by treating interference as Gaussian noise. We still want to constrain both the common and private rates of each transmitter so that this rate is utilized with as much "common rate" as possible. Specifically, let

$$a_i = \log\left(1 + \frac{\mathsf{SNR}_i^p}{1 + \mathsf{INR}_{ij}}\right)$$

be the required private rate at user i and

$$b_i = \log\left(1 + \frac{\mathsf{SNR}_i - \mathsf{SNR}_i^p}{1 + \mathsf{SNR}_i^p + \mathsf{INR}_{ij}}\right)$$

be the required common rate at user i. The rate a_i is the rate achieved by user i's private signal when treating the aggregate interference plus noise as Gaussian noise, while the rate b_i is the rate achieved by user i's common signal when treating its own private signal plus interference plus noise as Gaussian noise.

We say that (R_{ic}, R_{ip}) *fully utilizes the interference free rate* for user i if

$$R_{ip} \geq a_i \tag{43}$$

$$R_{ic} \geq b_i. \tag{44}$$

With this definition we have the following analogue of Lemma 6.

Lemma 14: If $(R_{1c}, R_{1p}, R_{2c}, R_{2p})$ fully utilizes the interference free rate for each user i, then there exists random common rates $R_{1r}, R_{2r} \geq 0$ such that $(R_{1c}, R_{1r}, R_{1p}, R_{1s}, R_{2c}, R_{2r}, R_{2p}, R_{2s})$ is self-saturated at both receivers for any choice of R_{1s}, R_{2s}.

Proof: This proof parallels exactly the proof of Lemma 6. We will show that one can always increase R_{2r} such that the overall sum rate constraint (the first constraint in (37)) is tight, so that receiver 1 is self-saturated. Suppose no such choice of R_{2r} exists. Then it must be that R_{2r} cannot be further increased because the second constraint in (37) is tight. However if this constraint is tight, then since $R_{1c} \geq b_1$, it can be seen that the first constraint must also be tight. ■

To complete the generalization of the deterministic case, we need a parallel result to Lemma 7, which we state next.

Lemma 15: For any point $(R_1, R_2) \in \mathcal{C}_{HK} \cap \mathcal{B}^-$ there exists a non-randomized Han-Kobayashi rate-split that fully utilizes the interference-free levels at each transmitter.

Proof: As in the deterministic case, we begin with an arbitrary non-randomized Han-Kobayashi rate-split and show that this can always be transformed into one that fully utilizes the interference free levels at each receiver. Note that again since $a_i + b_i = L_i$, for any point in $\mathcal{B}^-$ there will always be a sufficient amount of "rate" available to meet the constraints in (43) and (44).

First, we show that if $R_{ip} < a_i$ for either user i, then we can always increase R_{ip} and decrease R_{ic} by the same amount until $R_{ip} = a_i$. The only way such a transformation could not be done is if the second constraint in (37) prevented it. But by combining (38) with this constraint, it can be seen that this will never happen for $R_{ip} < a_i$. Moreover, since R_{ip} does not appear in any of the constraints in (37) at receiver j, such a change will never result in any of those constraints being violated.

Thus we can assume that $R_{ip} \geq a_i$. Given this, if a rate-pair does not fully utilize the interference free levels at transmitter i, it must be that $R_{ic} < b_i$. Suppose that this is true for receiver 1 and consider increasing R_{1c} and decreasing R_{1p} by the same amount until $R_{1c} = b_1$. Note that since $a_1 + b_1 = L_1$ and $R_{ic} + R_{ip} \geq L_1$, when we decrease R_{1p} in this way it will never cause it to become less than a_1. Changing R_{1p} and R_{1c} in this manner will not violate any of the generalized MAC constraints at receiver 1, since every constraint in (37) involving R_{1c} also involves R_{1p}. If this can be done without violating any constraints at receiver 2 then we are done. Otherwise, it must be that at least one of the constraints at receiver 2 involving R_{1c} becomes tight when R_{1c} reaches the value $R^*_{1c} = b_1 - \Delta$, for some $\Delta > 0$. The possible constraints here are the first and second. By the definition of $\mathcal{B}^-$ we have that $R_{2p}+R_{2c} \leq U_2 - 1$ and so

$$\begin{aligned} R_{2p} + R_{2c} + R^*_{1c} &= R_2 + R^*_{1c} \\ &\leq U_2 - 1 + b_1 - \Delta \\ &\leq \log(1 + \mathsf{SNR}_2 + \mathsf{INR}_{21}) - 1 - \Delta \qquad (45) \\ &\leq \log\left(1 + \frac{\mathsf{SNR}_2 + \mathsf{INR}_{21} - \mathsf{INR}^p_{12}}{\mathsf{INR}^p_{12} + 1}\right) - \Delta. \end{aligned}$$

This shows that the first constraint can not be tight. Note that the last inequality followed since $\mathsf{INR}^p_{12} \leq 1$. This implies that the second constraint must be tight, i.e.,

$$R_{2p} + R^*_{1c} = \log\left(1 + \frac{\mathsf{SNR}^p_2 + \mathsf{INR}_{21} - \mathsf{INR}^p_{21}}{\mathsf{INR}^p_{21} + 1}\right). \qquad (46)$$

Proceeding as in (45) we have

$$\begin{aligned} R_{2p} + R_{2c} + R_{1c}^{*} &\leq \log(1 + \mathsf{SNR}_2 + \mathsf{INR}_{21}) - 1 - \Delta \\ &\leq \log\left(1 + \frac{\mathsf{SNR}_2 + \mathsf{INR}_{21} - \mathsf{INR}_{12}^{p}}{\mathsf{INR}_{21}^{p} + 1}\right) - \Delta. \end{aligned}$$

Combining this with (46) we have

$$\begin{aligned} R_{2c} &\leq \log\left(1 + \frac{\mathsf{SNR}_2 - \mathsf{SNR}_2^{p} - \mathsf{INR}_{12}^{p}}{1 + \mathsf{SNR}_2^{p} + \mathsf{INR}_{21}}\right) - \Delta \\ &\leq b_2 - \Delta. \end{aligned}$$

And so it must also be that $R_{2p} \geq a_2 + \Delta$. Now consider increasing R_{1c} from R_{1c}^{*} by Δ, while simultaneously reducing R_{1p} and R_{2p} each by Δ and also increasing R_{2c} by Δ. As in the deterministic case, the above calculations show that we will not violate any of the constraints in the modified MAC regions at either receiver when doing this. After this transformation, the resulting rate-split will fully utilize the interference free levels at both receivers. ■

Combining the previous lemmas we have shown that all points in $\mathcal{C}_{HK} \cap \mathcal{B}^{-}$ are in $\mathcal{C}_{\mathrm{NE}}$, proving the first part of Theorem 2. Applying the next lemma will complete this proof by generalizing Lemma 15 for strong IC and showing that in that case the conclusions apply for all points in $\mathcal{C} \cap \mathcal{B}$.

Lemma 16: For a strong IC, any point $(R_1, R_2) \in \mathcal{C} \cap \mathcal{B}$ can be achieved by a non-randomized Han-Kobayashi rate split that fully utilizes the interference-free levels at each transmitter.

Proof: Recall, the for a strong interference channel, we set $\mathsf{INR}_{ij}^{p} = 0$ for each user i and so we have $a_i = 0$ and $b_i = L_i$. It follows that for any point $(R_1, R_2) \in \mathcal{C}_{HK} \cap \mathcal{B}$ there will be exactly one non-randomized Han-Kobayashi rate split that achieves this rate, namely the one with $R_{1c} = R_1$, $R_{2c} = R_2$ and $R_{1p} = R_{2p} = 0$. This will trivially fully utilize the interference-free levels at each transmitter. Furthermore, for such a channel $\mathcal{C}_{HK} = \mathcal{C}$, completing the proof. ■

As in the deterministic case, it can also be shown by direct calculation that $\mathcal{C}_{\mathrm{NE}}$ will always contain at least one point that is sum-rate optimal to within 1 bit.

V. Conclusions

We have formulated a new information theoretic notion of a Nash equilibrium region for interference channels. Moreover, we have used this notion to characterize the equilibria in both

deterministic and Gaussian ICs. In the deterministic case we are able to exactly specify the Nash equilibrium region, while in the Gaussian case we characterize the Nash equilibrium region to within one bit. The analysis for the Gaussian case directly parallels our analysis for the deterministic case, and thus serves as another illustration of the utility of deterministic models in providing useful insights for the more complicated Gaussian setting.

Our approach here is based on assuming that a given transmitter and the intended receiver share a source of common randomness. However, in the case of deterministic channels it is shown in [14] that this is not needed. Specifically, it is possible to achieve all points in $\mathcal{C} \cap \mathcal{B}$ by time-sharing among structured schemes which do no coding over time and use no common randomness. The key property of these structured schemes is that the common signals of the two users are *segregated* into separate levels at each of the receivers (in contrast to the random coding schemes considered in this paper, where the common signals of the two users are all mixed up.) Each transmitter may still use randomness to send a jamming signal on specified levels, but by aligning the jamming signal with the interfering common signal at the node's own receiver, the receiver does not need to decode it. Hence, the receiver needs not know the random bits generating the jamming signal. Such schemes can likely also be translated to the Gaussian settings by using structured codes instead of the Gaussian Han-Kobayashi schemes considered here.

The games we were considering here were games with full information, i.e., each user has perfect knowledge of all channel gains as well as the code-books of the other user. One possible future direction for this work would be to relax this assumption and consider games with incomplete information. Another natural direction would be to consider interference networks with more than 2 users. Some preliminary work in this direction for deterministic channels is given in [21] where it is shown that with more than two user efficient equilibria may no longer exist.

ACKNOWLEDGMENT

This work has benefited from convergence on a common game-theoretic formulation for interference channels that also appears in [22], which studies communication with secrecy constraints.

REFERENCES

[1] R. Etkin, D. Tse, and H. Wang, "Gaussian Interference Channel Capacity to within One Bit," *IEEE Trans. on Information Theory*, vol. 54, no. 12, pp. 5534-5562, 2008.

[2] T. Han and K. Kobayashi, "A new achievable rate region for the interference channel," *IEEE Trans. on Information Theory*, vol. 27, pp. 49-60, January 1981.

[3] G. Bresler and D. Tse, "The Two-User Gaussian Interference Channel: A Deterministic View," *European Transactions in Telecommunications*, vol. 19, pp. 333-354, April 2008.

[4] S. Avestimehr, S. Diggavi, and D. Tse, "Wireless Network Information Flow," in *Allerton Conference on Communication, Control, and Computing*, (Monticello, IL), pp. 16-22, September 2007.

[5] W. Yu and J. Cioffi, "Competitive Equilibrium in the Gaussian Interference Channel", *Proceedings of IEEE International Symposium on Information Theory,* pp. 431, 2000.

[6] S. T. Chung, S. J. Kim, J. Lee, and J.M. Cioffi, "A game-theoretic approach to power allocation in frequency-selective Gaussian interference channels," *Proceedings of IEEE International Symposium on Information Theory,* pp. 316, June 2003.

[7] A. Leshem, E. Zehavi , "Bargaining over the interference channel", *Proceedings of the IEEE International Symposium on Information Theory,* 2006.

[8] R. Etkin, A. P. Parekh and D. Tse, "Spectrum Sharing in Unlicensed Bands", *IEEE Journal on Selected Areas of Communication*, vol. 25, no. 3, pp. 517-528, April 2007.

[9] R. Myserson, *Game Theory: Analysis of conflict*, Harvard University Press, Cambridge MA, 1991.

[10] A. El Gamal and M. Costa, "The Capacity Region of a Class of Deterministic Interference Channels," IEEE Transactions on Information Theory, Vol. IT-28, No. 2, pp. 343-346, March 1982.

[11] R.G. Gallager, *Information Theory and Reliable Communication*, Wiley, 1968.

[12] H.F. Chong, M. Motani, H.K. Garg, and H. El Gamal, "On The Han–Kobayashi Region for the Interference Channel," *IEEE Transactions on Information Theory*, vol. 54, no. 7, pp. 3188-3194, 2008.

[13] R.G. Gallager, "A Perspective on Multiaccess Channels," *IEEE Transactions on Information Theory*, vol. 31, no. 2, pp. 124-142, March 1985.

[14] R. Berry and D. Tse, "Information Theoretic Games on Interference Channels," In *Proceeding of 2008 IEEE ISIT*, pp. 2518-2522, July 2008.

[15] X. Shang, G. Kramer and B. Chen, "A new outer bound and the noisy interference sum-rate capacity for Gaussian interference channels," *IEEE Transactions on Information Theory*, vol. 55, no. 2, pp. 689-699, 2009.

[16] V. Annapureddy and V. Veeravalli, "Sum capacity of the Gaussian interference channel in the low interference regime," *IEEE Transactions on Information Theory*, vol. 55, no. 7, pp. 3032-3050, 2009.

[17] A. Motahari and A. Khandani, "Capacity Bounds for the Gaussian Interference Channel," *IEEE Transactions on Information Theory*, vol. 55, no. 2, pp. 620-643, 2009.

[18] A. Carleial, "A case where interference does not reduce capacity," *IEEE Transactions on Information Theory*, vol. 21, no. 5, pp. 569-570, Sept. 1975.

[19] R. Berry and D. Tse, "Information Theory Meets Game Theory on The Interference Channel," In *Proceedings of the IEEE Information Theory Workshop (ITW)*, Volos, Greece, 2009.

[20] S. Diggavi and T. Cover, "Worst additive noise under covariance constraints," *IEEE Trans. on Information Theory*, vol. 47, pp. 3072-3081, Nov. 2001.

[21] R. Berry and S. Saha, “On Information Theoretic Interference Games With More Than Two Users,” In *Proceedings of the IEEE Information Theory Workshop (ITW)*, Cairo, Egypt, 2010.

[22] R. Yates, D. Tse and Z. Li, ”Secret Communication on Interference Channels”, In *Proc. of IEEE ISIT 2008*, pp. 374-378, 2008.